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NAME OF SUPERVISOR/*NOM DU DIRECTEUR DE THÈSE* Dr J.H. Masliyah

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THE UNIVERSITY OF ALBERTA

APPLICATION OF THE METHOD OF ORTHOGONAL COLLOCATION ON FINITE ELEMENTS TO ENGINEERING PROBLEMS

by

Ⓒ Dileep Kumar

A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH
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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled Application of the Method of Orthogonal Collocation on Finite Elements to Engineering Problems.

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in partial fulfilment of the requirements for the degree of Master of Science.

Masliyah
(Supervisor)

R.E. Mather

Ramon Bentsen

Date Dec 18, 1978

Abstract

The method of orthogonal collocation on finite elements (OCFE) was applied to two engineering problems. One of the problems considered is that of the flow of a Newtonian fluid in an internally finned tube. Cylindrical coordinates were employed and Legendre shifted orthogonal polynomials were used as the trial functions. An alternating direction implicit (ADI) method was used to solve the resulting set of equations. Better accuracy was achieved by increasing the number of collocation points per element rather than increasing the number of elements for a given number of interior collocation points. Although, in general, for a given total number of collocation points, the OCFE was found superior to the finite difference method in terms of accuracy, the computational time requirement was much higher for the method of orthogonal collocation on finite elements.

The second problem considered deals with the simulation of two-dimensional miscible displacement of oil by a solvent in porous media. A direct method of solution was used. Solution of the continuity equation provided excellent mass conservation. However, realistic concentration profiles could only be obtained from the convection diffusion equation for a very high value of the diffusion coefficient.

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Nomenclature

A	First derivative representation in the orthogonal collocation method
A_F	Dimensionless cross-sectional flow area (A'_F/R^2)
B	Second derivative representation in the orthogonal collocation method
C	Concentration of the solvent in the oil-solvent mixture
C_F	Dimensionless wetted perimeter (C'_F/R)
C_{in}	Source concentration
dV	Differential volume, cm^3
f	Fanning fraction factor
f.Re	Product of Fanning friction factor and Reynolds number
F	$= Z\frac{\Delta x_k}{\Delta y_\ell}$, a dimensionless quantity
g	Dimensionless variable
j_f	Collocation point at fin tip
K_D	Dispersion Coefficient, cm^2/s ($K_D = \phi K_{DT}$)
K_{DT}	Total Dispersion Coefficient, cm^2/s
K_P	Permeability, darcy
k_1, k_2, k_3 and k_4	k-values of the fourth order Runge-Kutta method
L'	Dimensional length, cm
L	Fin length, dimensionless (Chapter 4)
	Length of the formation, cm (Chapter 5)
ℓ_f	Fin tip element
N	Number of interior collocation points in one direction per element
NE	Number of elements

NF	Number of fins
NR, NX and Nθ	Total number of collocation points (including boundary collocation points for GOC in r, x and θ directions, respectively
NEθ, NER, NEX and NEY	Number of elements in θ, r, x and y directions respectively
NPθ, NPR, NPX and NPY	Number of collocation points per element in θ, r, x and y directions respectively (=N+2)
P, p'	Pressure, psi
P_i	Orthogonal polynomial, Legendre shifted polynomial
$P_{i,j}^{k,\ell}$	Pressure at collocation point (x_i,y_j) in the $k\ell^{th}$ element
q, $q(x',y')$	Source term, cm^3/cm^3 formation/sec
Q	Source term, cm^3/s
Q_C	Calculated Q using a quadrature approach
R	Tube radius, cm
r'	Radial coordinate
r	Dimensionless radial coordinate (r'/R)
r_j	Value of r at collocation point (θ_i, r_j)
Re	Reynolds number
S	Thickness of the formation, cm
t	Time, s
Δt	Increment in t, s
$\underline{u}$	Darcy velocity, cm/s
u_x, u_y	Component of the Darcy velocity in x and y directions, cm/s
$u_{x_{i,j}}^{k,\ell}$ or $u_x^{k,\ell}(x_i,y_j)$	x-component of the velocity of collocation point (x_i,y_j) in the $k\ell$th element

w, w_i	Weighting factor, weighting factor in the i^{th} row
W	Width of the formation, cm
W'	Dimensional velocity, cm/s
$W^{k,\ell}(\theta,r)$	Dimensionless velocity
W_c	Centre velocity, dimensionless
$\langle W \rangle$	Average velocity, dimensionless
$W^{k,\ell}_{i,j}$	Velocity at collocation point (θ_i, r_j) in the $k\ell^{th}$ element, dimensionless
WF	A weighting factor
x'	Axial direction, cm
x	axial direction dimensionless
Δx_k	$k\ell^{th}$ element size in x-direction
y	Transverse direction dimensionless
Δy_ℓ	$k\ell^{th}$ element size in y direction
Z	A constant

Greek Symbols

α	Semiangle of fin separation
θ	Angular coordinate
ϕ	Porosity
$\mu, \mu^{k,\ell}_{i,j}$	Viscosity, viscosity at collocation point (x_i, y_j) in $k\ell^{th}$ element, cp
ω	Iteration parameter
Λ	Solution domain
ν	Dimensionless variable

Subscripts

i,j,k,ℓ and n	indices
O	Oil
s	Solvent
f	Fin
x,y	In x and y directions
-	Vector

Superscripts

-	Vector
=	Matrix
'	Denotes a dimensionless quantity
k,ℓ	Denotes $k\ell^{th}$ element
s, $s+\frac{1}{2}$, $s+1$	Denotes initial value, value after one half iteration and value after one complete iteration, respectively
$t, t+\Delta t$	Denotes value after t and $t+\Delta t$ time levels respectively

Chapter 1

INTRODUCTION

There has been a growing interest in the application of the method of orthogonal collocation on finite elements (OCFE) to a wide variety of chemical engineering problems. The OCFE method combines the features of the orthogonal collocation method with those of the finite element method. The main advantage of the orthogonal collocation method is its rapid convergence. In the finite element method, one can change shape or size of the elements to fit irregular boundaries. Moreover, additional elements can be inserted in the regions of steep gradients.

The OCFE method has been shown to be very powerful in obtaining solutions to one-dimensional problems. However, not enough work has been done to demonstrate the suitability of the method to multidimensional problems.

The purpose of the present work is to study the suitability of the recently developed OCFE method to two-dimensional problems. In order to gain confidence in the method, it was first applied to a simple one-dimensional problem. The program listing for the one-dimensional problem is given in Appendix C.

The applicability of the OCFE method to fluid flow in finned tubes is discussed in Chapter 4. The problem has a steep gradient at one of the boundaries. The reason for the choice of this flow problem is that the numerical techniques which were used to solve the problem could not handle the steep gradient unless a large number of grid points were used. The resulting algebraic equations were solved using the ADI technique.

The applicability of the OCFE method to porous media is discussed in Chapter 5. The problem deals with the unsteady state miscible displacement of oil by solvent in an oil reservoir. Unlike the finned tube problem, a direct method of solution was used. The direct method solved all the equations simultaneously using the LU decomposition technique with iterative refinement. The direct method was employed because the ADI technique was found to be fairly expensive for the finned tube problem. The two different locations for the production well were considered. In one of the schemes, the geometry represented a quarter of a five spot. The second scheme which has no practical importance was used mainly to check the numerical results.

CHAPTER 2

LITERATURE REVIEW

The method of weighted residuals encompasses several methods (Subdomain, Collocation, Galerkin etc.). These methods were first unified by Crandall (1956) as the method of weighted residuals (MWR). A comprehensive review of the literature on MWR is available in Finlayson (1972, 1974).

The method of weighted residuals was applied to a wide variety of engineering problems by Clymer and Braun (1973), Finlayson and Scriven (1966) and Vichnevetsky (1969). Application of the Galerkin method to reservoir engineering was considered by Cavendish et al. (1969), Culham and Varga (1971) and McMichael and Thomas (1973). Cavendish et al. described a new technique based on the Galerkin method which used both high and low order piecewise polynomial approximations to solve boundary value problems in reservoir engineering. Culham and Varga, in addition to the Galerkin method, applied non-Galerkin cubic spline interpolation to solve non-linear parabolic equations. McMichael and Thomas investigated the feasibility of using the Galerkin method on three phase multidimensional compressible flow. They observed that for a given time step the Galerkin method required much more computer time than the finite difference model. However, the Galerkin method was capable of handling a larger time step.

A collocation method was first applied by Slater (1934) to solve differential equations. Frazer et al. (1937) used this method with various trial functions. Lanczos (1938) used the roots to Tchebychev polynomials as collocation points. Sparrow and Haji-Sheikh

(1968) successfully applied a least square collocation method to steady state heat conduction in arbitrary bodies. The first known application of a boundary collocation method is due to Sparrow and Loeffler (1959).

The method of orthogonal collocation was first applied by Lanczos (1938, 1956). It has since been applied by Cleanshaw and Norton (1963), Norton (1964), and Wright (1964) to solve ordinary differential equations. Villadsen and Stewart (1967) applied the orthogonal collocation method to boundary value problems. The method of orthogonal collocation has been shown to be very effective for certain non-linear chemical engineering problems and has been highly advocated by Finlayson (1971), Young and Finlayson (1973).

Sincovec (1977) described the development of a generalized collocation method for the solution of coupled non-linear parabolic partial differential equations. He showed that the collocation method with Gaussian collocation points was more effective than the conventional finite difference solution. He also showed that for problems with a smooth solution, one would obtain more accuracy per unit time by increasing the order of the collocation method.

The method of orthogonal collocation on finite elements (OCFE) which is the subject of this thesis is a rather new technique. The area is not well explored and not much work has been done on this method. Douglas and Dupont (1973) studied theoretically a finite element collocation method for parabolic equations. Bladier (1973) used OCFE to solve a die swell problem unsuccessfully. Anderman (1974) used this method to solve a two dimensional fluid flow around a sphere at a very low Reynolds number. He found that the computational time requirement was very large. Carey and Finlayson (1975) used the OCFE method to solve a one dimensional effectiveness factor problem in a catalyst pellet and they highly recommended the use of OCFE. Chang and Finlayson (1977) applied the OCFE method to a two dimensional problem and used the alternating direction implicit (ADI) method to solve the resulting algebraic equations.

CHAPTER 3

THE METHODS OF WEIGHTED RESIDUALS AND ORTHOGONAL COLLOCATION ON FINITE ELEMENTS

3.1 General Treatment:

The method of weighted residuals (MWR) is a general method of obtaining solutions to differential equations. The solution to be determined is expanded in a set of specified trial functions. The constants of the trial functions are obtained using MWR. The first approximation gives a solution within 20%. However, more accurate solutions can be obtained using higher approximations.

For illustrative purposes, a boundary value problem is considered, Finlayson (1972).

$$\nabla^2 T = T_{xx} + T_{yy} = 0 \quad \text{in } V(x,y) \tag{3.1}$$

$$T = T_o \qquad \text{on the boundary of } V \tag{3.2}$$

Assuming a trial function of the form

$$T = T_o + \sum_{i=1}^{n} C_i T_i \tag{3.3}$$

where functions T_i satisfy the boundary conditions ($T_i = 0$ on the boundary). Substitute Equation (3.3) in Equation (3.1) to form the residual (The residual is zero everywhere in V when the trial function is the exact solution).

$$R = \nabla^2 (T_o + \sum_{i=1}^{N} C_i T_i) \tag{3.4}$$

or

$$R = \nabla^2 T_o + \sum_{i=1}^{N} C_i \, \nabla^2 T_i$$

In the method of weighted residuals, C_i are chosen in such a way that the residual is forced to be zero in an average sense.

Equating the weighted integrals of the residual to zero yields

$$(WF_j, R) = 0 \qquad j = 1,2\ldots\ldots n \tag{3.5}$$

where $$(WF_j, R) = \int_V WF_j \; R \; dV \tag{3.6}$$

and WF is a weighting factor. When WF and R are orthogonal then

$$\int_V WF \; R \; dV = 0$$

From Equations (3.4) and (3.5) one obtains,

$$\sum_{i=1}^{N} C_i (WF_j, \nabla^2 T_i) = - (WF_j, \nabla^2 T_o) \tag{3.7}$$

Equation (3.7) can be written as

$$\sum_{i=1}^{N} G_{ji} C_i = H_j \tag{3.8}$$

where $G_{ji} = (WF_j, \nabla^2 T_i)$

$H_j = - (WF_j, \nabla^2 T_o)$

Here T_o and T_i are known. Therefore G_{ji} and H_j can be evaluated if WF_j is known. The methods of choosing WF_j are described below. Once G_{ji} and H_j are known, C_i can be evaluated using Equation (3.8). C_i can then be substituted in Equation (3.3) to obtain the approximate solution.

There are various ways of choosing the weighting functions WF. Each choice provides a different method of weighted residuals. Some of the important methods are considered below.

a) Subdomain method: Dividing the domain V into n_D smaller subdomains V_j and defining

$$WF_j = \begin{cases} 1 & \underline{X} \text{ in } V_j \\ 0 & \underline{X} \text{ not in } V_j \end{cases}$$

One observes that the residual R, of the differential equation when integrated over the subdomain, V_j, is zero as given by Equation (3.5). As the number of subdomains increase, the differential equation is satisfied in more and more subdomains and the residual approaches zero everywhere in the limit as $n_D \rightarrow \infty$.

b) Least squares method: In the least squares method the weighting function is $\frac{\partial R}{\partial C_i}$. Equating the residual to zero one obtains,

$$\int_V \frac{\partial R}{\partial C_i} R \, dV = 0 \tag{3.9}$$

or $$\frac{\partial}{\partial C_i} \int_V R^2 \, dV = 0 \quad \text{for } i = 1,2...n \tag{3.10}$$

Hence the integral is minimized with respect to C_i. Solution of Equation (3.10) provides the C_i coefficients. The algebra involved using this method is usually rather tedious.

c) Galerkin Method: In the Galerkin method, the weighting functions are also the trial functions, i.e., $WF_i = T_i$. The trial functions must be part of a complete set of functions so that the trial solution is capable of representing the exact solution provided enough terms are used. The Galerkin method forces the residual to be zero by making it orthogonal to each member of a complete set of functions.

d) Method of Moments: In this method the weighting functions for the one dimensional case are $1, x, x^2, x^3, \ldots$ Therefore successively higher moments of the residuals are forced to be zero. It is evident that for the first approximation this method is identical to the subdomain method.

e) Collocation Method: In the collocation method the weighting functions are the displaced Dirac delta function

$$WF_j = \delta(x - x_j) \tag{3.11}$$

which has the property that

$$\int_V WF_j \; R \; dV = R|_{x_j} \tag{3.12}$$

Thus the residual is zero at the N collocation points x_j and it approaches zero everywhere in the limit as $N \to \infty$.

It has been shown that in the collocation method the solution depends upon the choice of the collocation points at which the residual is set to zero, Finlayson (1972). In order to reduce such dependence one can apply the least squares collocation method. In this method the residual is evaluated at more points than there are coefficients and the over-determined set of algebraic equations are solved by a least squares method.

The orthogonal collocation method which is a special case of the collocation method is discussed in Section 3.3.

3.2 Choice of Trial Functions:

One of the most important considerations in using MWR is the choice of the trial functions. Such a choice is very important for low order approximations but for higher order approximations it is not as critical since the rate of convergence becomes the prime criterion, Finlayson (1972).

The trial functions must be complete so that they represent the exact solution if enough terms are used. Further the trial function should be as simple as possible and should not complicate the analysis unnecessarily. For a problem with a boundary condition of the type $y(x,z) = F(x,z)$ one may choose

$$y(x,z) = F(x,z) + \sum_{i=1}^{N} a_i \, y_i \, (x,z)$$

where it is specified that $y_i = 0$ on the boundary. Thus the choice satisfies the boundary condition. One may start with a general polynomial and can obtain a reasonable trial function after applying boundary conditions and symmetry conditions. Orthogonal polynomials were found to be excellent trial functions (Finlayson 1972) and can be constructed to satisfy some of the boundary conditions. This approach is usually used in the orthogonal collocation method.

3.3 Method of Orthogonal Collocation:

The method of orthogonal collocation has been well covered by Finlayson (1972). However, a brief description is presented in this section in order to facilitate the understanding of the method which was applied to the two dimensional problems discussed in this thesis.

The advantage of the method of orthogonal collocation is the rapid convergence to the solution as the number of collocation points is increased. Ferguson and Finlayson (1972) showed that for an ordinary differential equation the error was proportional to $(\frac{1}{N})^{1.72N}$ where N is the number of interior collocation points. As N changes from 5 to 6 the error decreases by a factor of 100. In the finite difference calculation of $O(\Delta x^2)$, a change of N $(= \frac{1}{\Delta x})$ from 5 to 6 decreases the error by only a factor of 1.4.

In the orthogonal collocation method the collocation points are taken as the roots to orthogonal polynomials. Villadsen and Stewart (1967) chose the trial functions to be the sets of orthogonal polynomial which also satisfied the boundary conditions and the roots to the polynomials gave the collocation points.

A brief description of the orthogonal collocation method is presented below.

Let a function t be approximated by a trial function,

$$t(x) = b_1 + b_2\, x + x\,(1-x) \sum_{i=1}^{NX-2} a_i P_{i-1}\,(x) \qquad (3.13)$$

where the polynomials, P_i, are defined by

$$\int_0^1 WF(x)\; P_n(x)\; P_m(x)\; dx = 0 \qquad (3.13a)$$

$$n = 0,\ 1, \ldots .m-1$$

$$\text{and } m \neq n$$

Thus the successive polynomials are orthogonal to all polynomials of order less than m with some weighting function $WF(x) \geq 0$. NX is the total number of collocation points (including boundary points). Since both even and odd powers of x are included in the trial functions, it is clear that the orthogonal polynomials have no special symmetry properties. Equation (3.13) can be substituted in the differential equation whose solution is required and the residuals are set to zero at the given collocation points in the interval (0,1). Consequently the coefficients of Equation (3.13) can be evaluated. The collocation points are the roots to the polynomial $P_n(x)$. Finlayson's approach is not to solve for the constants but for t at the collocation point. Equation (3.13) can be written as,

$$t(x) = \sum_{i=1}^{NX} d_i \, x^{i-1} \tag{3.14}$$

Taking first and second derivatives at the collocation points, one obtains

$$t(x_j) = \sum_{i=1}^{NX} d_i (x_j)^{i-1} \tag{3.15}$$

$$\left.\frac{dt}{dx}\right|_{x_j} = \sum_{i=1}^{NX} \frac{d}{dx} x^{i-1} \Big._{x_j} d_i = \sum_{i=1}^{NX} (i-1)\,(x_j)^{i-2}\, d_i \tag{3.16}$$

$$\left.\frac{d^2t}{dx^2}\right|_{x_j} = \sum_{i=1}^{NX} \frac{d^2}{dx^2} x^{i-1} \Big._{x_j} d_i = \sum_{i=1}^{NX} (i-1)(i-2)(x_j)^{i-3}\, d_i \tag{3.17}$$

where NX=N+2. N is the total number of interior collocation points. Writing the above equations in the matrix notation yields,

$$\overline{t} = \overline{\overline{Q}}\,\overline{d} \tag{3.18}$$

$$\overline{\frac{dt}{dx}} = \overline{\overline{C}}\,\overline{d} \tag{3.19}$$

and

$$\overline{\frac{d^2t}{dx^2}} = \overline{\overline{D}}\,\overline{d} \tag{3.20}$$

where

$$Q_{ji} = x_j^{i-1} \tag{3.21}$$

$$C_{ji} = (i-1)\, x_j^{i-2} \tag{3.22}$$

$$D_{ji} = (i-1)(1-2)\, x_j^{i-3} \tag{3.23}$$

From equation (3.18)

$$\bar{d} = \bar{\bar{Q}}^{-1} \, \bar{t} \qquad (3.24)$$

Substituting $\bar{d}$ in Equations (3.19) and (3.20) yields

$$\overline{\frac{dt}{dx}} = \bar{\bar{C}} \, \bar{\bar{Q}}^{-1} \, \bar{t} \equiv \bar{\bar{A}} \, \bar{t} \quad \text{where } \bar{\bar{A}} = \bar{\bar{C}} \, \bar{\bar{Q}}^{-1} \qquad (3.25)$$

$$\frac{d^2t}{dx^2} = \bar{\bar{D}} \, \bar{\bar{Q}}^{-1} \, \bar{t} \equiv \bar{\bar{B}} \, \bar{t} \quad \text{where } \bar{\bar{B}} = \bar{\bar{D}} \, \bar{\bar{Q}}^{-1} \qquad (3.26)$$

As the collocation points x_j are known, matrices $\bar{\bar{Q}}$, $\bar{\bar{C}}$ and $\bar{d}$ can be evaluated and hence $\bar{\bar{A}}$ and $\bar{\bar{B}}$ can be determined. A general computer program to calculate matrices $\bar{\bar{Q}}$, $\bar{\bar{C}}$, $\bar{d}$, $\bar{\bar{A}}$ and $\bar{\bar{B}}$ is provided in Appendix A. The matrices are tabulated in Appendix B for $3 \leq NX \leq 10$. Tabulation of matrices A and B is also given in Finlayson (1972) for NX = 3 and 4.

The derivatives in a differential equation whose solution is to be found can be replaced by Equations (3.25) and (3.26). The resulting set of algebraic equations can be solved subject to the prevailing boundary conditions. For a differential equation of the type

$$\frac{d^2t}{dx^2} + \frac{dt}{dx} + x\,t = 0 \qquad (3.27)$$

one obtains

$$\sum_{i=1}^{NX} B_{j,i} \, t_i + \sum_{i=1}^{NX} A_{j,i} t_i + x_j \, t_j = 0 \qquad (3.28)$$

for each interior collocation point j.

3.4 Orthogonal Collocation for Two-Dimensional Problems:

For two-dimensional problems a trial function analogous to

Equation (3.13) is given by, Finlayson (1974).

$$T(x,y) = \left\{ b_1+b_2 x + x(1-x) \sum_{k=1}^{NX-2} a_k P_{k-1}(x) \right\} \quad x$$
$$\left\{ C_1+C_2 y + y(1-y) \sum_{\ell=1}^{NY-2} F_\ell P_{\ell-1}(y) \right\} \tag{3.29}$$

or

$$T(x,y) = \left\{ \sum_{k=1}^{NX} d_{x,k}\, x^{k-1} \right\} \left\{ \sum_{\ell=1}^{NY} d_{y,\ell}\, y^{\ell-1} \right\} \tag{3.30}$$

where

$$d_{x,k} = \left[\bar{d}_x \right]_k \tag{3.31}$$

$$d_{y,\ell} = \left[\bar{d}_y \right]_\ell \tag{3.32}$$

For any particular point (i, j) one can write

$$T(x_i,y_j) = \left\{ \sum_{k=1}^{NX} d_{x,k}\, x_i^{k-1} \right\} \left\{ \sum_{\ell=1}^{NY} d_{y,\ell}\, y_j^{\ell-1} \right\} \tag{3.33}$$

Defining matrix $\bar{\bar{T}}$ by $T_{i,j} = T(x_i,y_j)$ Equation (3.33) becomes

$$T_{ij} = \left\{ \bar{\bar{Q}}_x\, \bar{d}_x \right\}_i \left\{ \bar{\bar{Q}}_y\, \bar{d}_y \right\}_j \tag{3.34}$$

where Q_x and Q_y are $\bar{\bar{Q}}$ in x and y directions respectively and their values are given by Equation (3.21) or in the matrix form by

$$\bar{\bar{T}} = \left[\bar{\bar{Q}}_x \bar{d}_x \right] \left[\bar{\bar{Q}}_y \bar{d}_y \right]' \tag{3.35}$$

or

$$\bar{\bar{T}} = \bar{\bar{Q}}_x\, \bar{d}_x\, \bar{d}_y'\, \bar{\bar{Q}}_y' \tag{3.36}$$

The first derivative with respect to x is given by

$$\frac{\partial \overline{\overline{T}}}{\partial x} = \overline{\overline{C}}_x \, \overline{d}_x \, \overline{d}_y{}' \, \overline{\overline{Q}}_y{}' \tag{3.37}$$

$$= \overline{\overline{C}}_x \left[\overline{\overline{Q}}_x{}^{-1} \, \overline{\overline{T}} \right]$$

$$= \overline{\overline{C}}_x \, \overline{\overline{Q}}_x{}^{-1} \, \overline{\overline{T}}$$

or

$$\frac{\partial \overline{\overline{T}}}{\partial x} = \overline{\overline{A}}_x \, \overline{\overline{T}} \tag{3.38}$$

or

$$\frac{\partial T_{i,j}}{\partial x} = \sum_k A_{i,k} \, T_{k,j} \tag{3.39}$$

The first derivative with respect to y is given by

$$\overline{\overline{T}} = \overline{\overline{Q}}_x \, \overline{d}_x \, \overline{d}_y{}' \, \overline{\overline{Q}}_y{}'$$

$$\frac{\partial \overline{\overline{T}}}{\partial y} = \left(\frac{\partial \overline{\overline{T}}'}{\partial y} \right)' \tag{3.40}$$

where

$$\overline{\overline{T}}' = \overline{\overline{Q}}_y \, \overline{d}_y \, \overline{d}_x \, \overline{\overline{Q}}_x{}' \tag{3.41}$$

and

$$\frac{\partial \overline{\overline{T}}'}{\partial y} = \overline{\overline{C}}_y \, \overline{d}_y \, \overline{d}_x{}' \, \overline{\overline{Q}}_x{}' \tag{3.42}$$

$$= \overline{\overline{C}}_y \, \overline{\overline{Q}}_y{}^{-1} \, \overline{\overline{T}}' \tag{3.43}$$

or

$$\frac{\partial \overline{\overline{T}}'}{\partial y} = \overline{\overline{A}}_y \, \overline{\overline{T}}' \tag{3.44}$$

As the same collocation points are being used in both x and y directions, the subscripts x and y from A, B, C, d and Q can be dropped.

$$\text{Let } \overline{\overline{Z}} = \overline{\overline{T}}' \tag{3.45}$$

hence

$$Z_{i,j} = T_{j,i}$$

Equation (3.43) gives

$$\frac{\partial Z_{i,j}}{\partial y} = \sum_k A_{i,k} Z_{k,j} \tag{3.46}$$

Interchanging i and j in Equation (3.46) yields

$$\frac{\partial Z_{j,i}}{\partial y} = \sum_k A_{j,k} Z_{k,i} \tag{3.47}$$

or

$$\frac{\partial T_{i,j}}{\partial y} = \sum_k A_{j,k} T_{i,k} \tag{3.48}$$

Similar relations hold for the second derivatives.

Thus for a two dimensional problem of the type

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = f(x,y) \tag{3.49}$$

one obtains

$$\sum_{n=1}^{NX} B_{i,n} T_{n,j} + \sum_{n=1}^{NY} B_{j,n} T_{i,n} = f(x_i,y_j) \tag{3.50}$$

for any interior collocation point (i,j)

3.5 The Method of Orthogonal Collocation on Finite Elements (OCFE)

The main difference between the method of weighted residuals (MWR) and the traditional finite element method is in the choice of the trial functions. Normally MWR uses trial functions defined over the entire domain whereas in the traditional finite element method trial or shape functions are defined over each element. The advantage of the finite element method is that the elements can be changed in shape or size to fit the physical boundaries. The method of orthogonal collocation on finite elements is an attempt to combine the features of both the orthogonal collocation method and the finite element method.

The main feature of OCFE is that the domain of interest is divided into subdomains and that the trial function is applied over the domain in a piecewise fashion element by element. Using such discretization OCFE should then be able to handle solutions which have steep gradients. As the trial functions are orthogonal in the region (0,1) it becomes necessary to have the independent variable(s) to lie between (0,1). For a given element, the value of the residuals give an indication as to whether more elements need to be added in a given region. The total number of iterations can be reduced by using a solution obtained with a lesser number of collocation points as the initial guess for a higher number of collocation points. Irrespective of the nature of the solution, (symmetric or unsymmetric), a general polynomial approximation should be used in OCFE. A given differential equation is satisfied at each interior collocation point. At the element interboundaries, continuity of the first derivative is sought. For a one-dimensional problem, one obtains a block diagonal matrix. Two-dimensional problems can be solved as one-dimentional problems using the alternating direction implicit method (ADI).

Appendix C illustrates the solution of a simple one-dimensional problem using OCFE. The computer program is also included. A similar approach was used to solve the two-dimensional fin problem discussed in Chapter 4.

3.5.1. Approximation Errors:

Douglas and Dupont (1973) showed that, for parabolic differential equations the discretization error was proportional to h^4 ($h=1/NE$) for $N=2$, when the collocation points were the Gaussian quadrature points. However, when the collocation points were uniformly distributed in each element, the error was proportional to h^2. Thus changing the collocation points to Gaussian quadrature points reduces the error dramatically. More generally, Deboor and Swartz (1973) showed that for a differential equation of the type $D^2y = f(y)$, the error for the OCFE could be given by the following relation.

$$\text{error} \propto \left(\frac{1}{NE}\right)^{N+2}$$

Douglas (1973) showed that for linear problems, when the trial polynomials were of degree (N+1), convergence proceeded as Δx^{n+2} globally and Δx^{2n} at the collocation points.

CHAPTER 4

APPLICATION OF THE METHOD OF ORTHOGONAL COLLOCATION ON FINITE ELEMENTS TO FINNED TUBES

The method of orthogonal collocation was applied to a difficult fluid flow problem. The selected physical problem is that of an incompressible Newtonian fluid flow in the internally finned tube shown in Figure 1. The governing equation is an elliptic type of partial differential equation. This problem was chosen mainly because traditional finite difference techniques did not provide an adequate solution unless a large number of grid points were used and other techniques which are usually used to solve these type of equations failed to produce a satisfactory solution.

4.1 Statement of the Problem:

The momentum equation governing the fluid flow is given by:

$$\frac{1}{r'}\frac{\partial}{\partial r'}\left\{r'\frac{\partial W'}{\partial r'}\right\} + \frac{1}{r'^2}\frac{\partial^2 W'}{\partial \theta^2} = \frac{1}{\mu}\frac{dP'}{dx'} \tag{4.1}$$

Introducing the dimensionless quantities,

$$r = \frac{r'}{R} \quad \text{and} \quad W = \frac{-W'}{\frac{R^2}{\mu}\left(\frac{dP'}{dx'}\right)}$$

The flow equation becomes

$$\frac{\partial^2 W}{\partial r^2} + \frac{1}{r}\frac{\partial W}{\partial r} + \frac{1}{r^2}\frac{\partial^2 W}{\partial \theta^2} = -1 \tag{4.2}$$

The appropriate boundary conditions are (Figure 1)

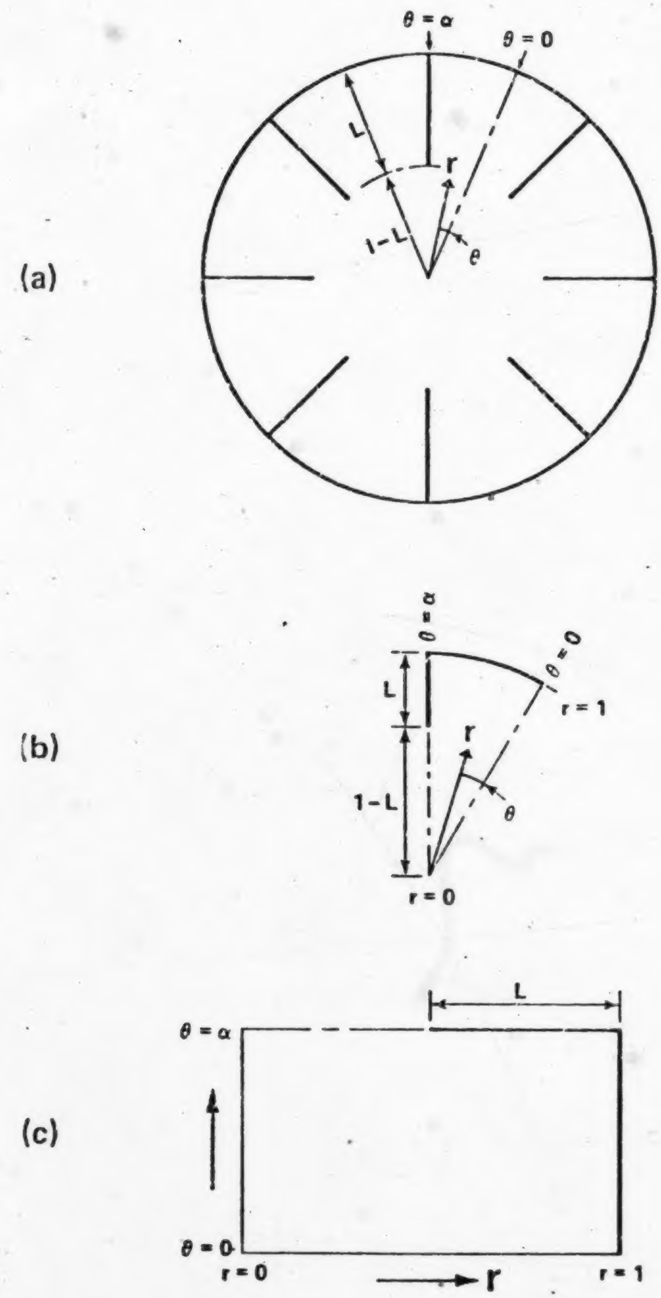


FIGURE 1. Flow Geometry of a Finned Tube

$$W = 0 \quad \text{at } r = 1 \text{ and } 0 \le \theta \le \alpha \tag{4.3a}$$

$$W = 0 \quad \text{at } \theta = \alpha \text{ and } 1\text{-}L \le r \le 1 \tag{4.3b}$$

$$\frac{\partial W}{\partial \theta} = 0 \quad \text{at } \theta = 0 \text{ and } 0 < r < 1 \tag{4.3c}$$

$$\text{at } \theta = \alpha \text{ and } 0 < r < 1\text{-}L \tag{4.3d}$$

$$\frac{\partial W}{\partial r} = 0 \quad \text{at } r = 0 \text{ and all } \theta \tag{4.3e}$$

Due to the complexity of the boundary conditions and the presence of a sudden velocity variation at the fin tip, it is unlikely that a closed form analytical solution exists.

4.2 OCFE Formulation of the Problem:

For every $k\ell^{th}$ element (Figure 2) two new variables are defined such that

$$g^k = \frac{\theta-\theta_k}{\Delta\theta_k}$$

$$\nu^\ell = \frac{r-r_\ell}{\Delta r_\ell} \tag{4.4}$$

where

$$\Delta\theta_k = \theta_{k+1} - \theta_k \tag{4.5}$$

and

$$\Delta r_\ell = r_{\ell+1} - r_\ell \tag{4.6}$$

Thus in any $k\ell^{th}$ element both the variables g and ν vary from zero to one.

OCFE is applied at each interior collocation point of each element ($k\ell$). Equation 4.2 can be written as:

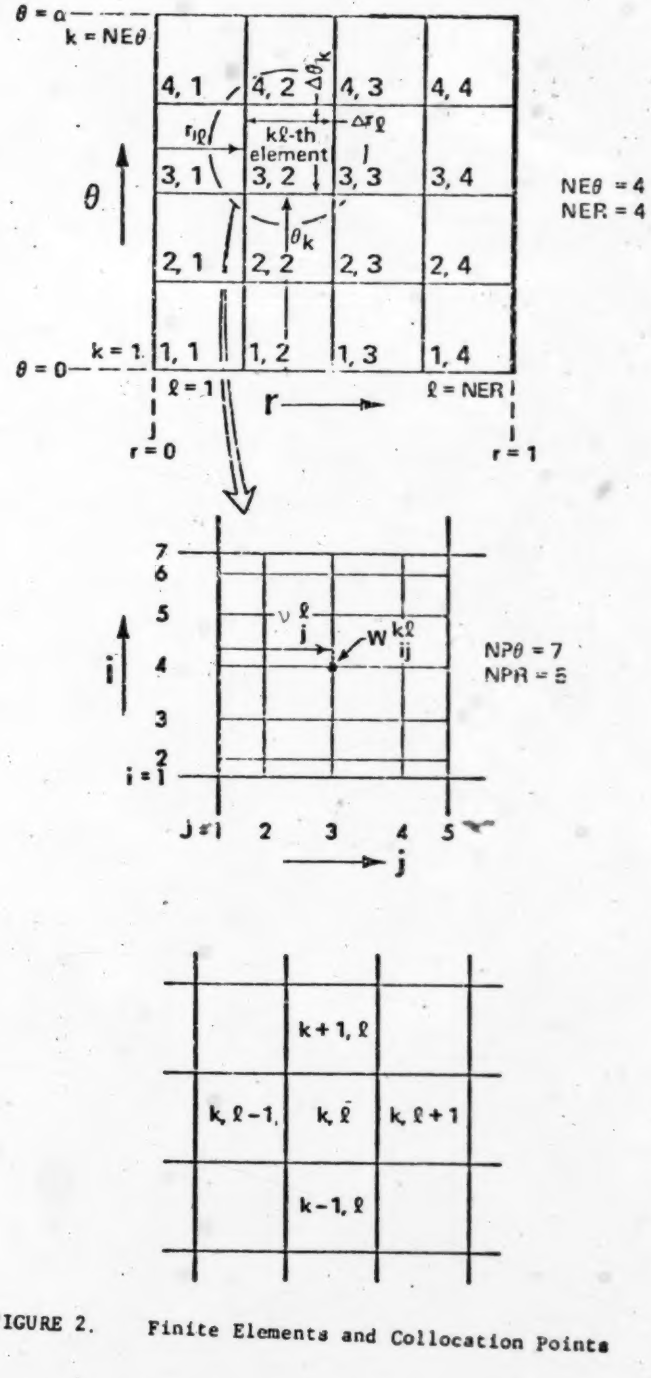


FIGURE 2. Finite Elements and Collocation Points

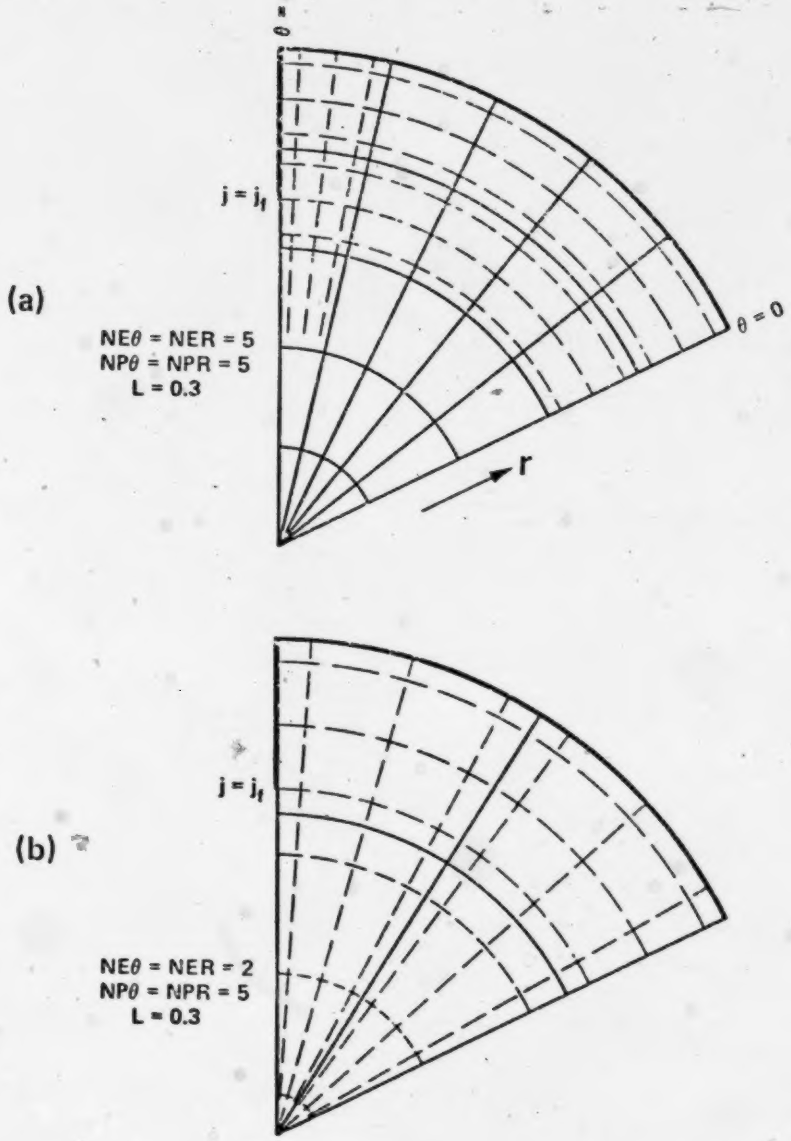


FIGURE 3 Collocation Points Near the Fin Tip

$$\frac{1}{\Delta r_\ell^{2}} \sum_{n=1}^{NPR} B_{j,n} W_{k,n}^{k,\ell} + \frac{1}{(r_\ell + \nu_j^\ell \Delta r_\ell)\, \Delta r_\ell} \sum_{n=1}^{NPR} A_{j,n} W_{i,n}^{k,\ell}$$

$$+ \frac{1}{(\Delta\theta_k)^2 (r_\ell + \nu_j^\ell \Delta r_\ell)^2} \sum_{n=1}^{NP\theta} B_{i,n} W_{n,j}^{k,\ell} = -1 \qquad (4.7)$$

where $W_{i,j} = W(\theta_i,\ r_j)$ (4.8)

and

$$\begin{aligned} i &= 2, \ldots\ldots, NP\theta-1 \\ j &= 2, \ldots\ldots, NPR-1 \\ k &= 1, \ldots\ldots, NE\theta \\ \ell &= 1, \ldots\ldots, NER \end{aligned} \qquad (4.9)$$

For a given element $k\ell$, $NP\theta$ and NPR are the total number of collocation points (including boundary points) in θ and r directions, respectively. $NE\theta$ and NER are the number of elements in θ and r directions, respectively. Figure 2 shows the elements on which Equation (4.7) is applied. Essentially Equation (4.7) is applied to each element of the flow field. On the element interboundaries continuity of the function and its first derivative is assumed. For the element boundaries that coincide with the tube boundaries, the physical boundary conditions as given by Equation (4.3) are applied. The algebraic Equations (4.7) together with the boundary conditions are solved using the ADI method, Peaceman and Rachford (1955).

4.3 Solution of the Equations:

Equation (4.7) can be written as

$$\left\{\frac{r_\ell+\nu_j^\ell\ \Delta r_\ell}{\Delta r_\ell}\right\}^2 \sum_{n=1}^{NPR} B_{j,n}\, W_{i,n}^{k,\ell} + \left\{\frac{r_\ell+\nu_j^\ell\ \Delta r_\ell}{\Delta r_\ell}\right\} \sum_{n=1}^{NPR} A_{j,n}\, W_{i,n}^{k,\ell}$$

$$+ \frac{1}{\Delta\theta_k^2} \sum_{n=1}^{NP\theta} B_{i,n}\, W_{n,j}^{k,\ell} = -\,(r_\ell+\nu_j^\ell\ \Delta r_\ell)^2 \qquad (4.10)$$

To solve the system of equations using ADI, Equation (4.10) is written in two different forms.

4.3.1 Constant r solution:

For constant r, Equation (4.10) can be written as.

$$\omega\, W_{i,j}^{k,\ell,s+\frac{1}{2}} - \frac{1}{\Delta\theta_k^2} \sum_{n=1}^{NP\theta} B_{i,n}\, W_{n,j}^{k,\ell,s+\frac{1}{2}} = \omega\, W_{i,j}^{k,\ell,s}$$

$$\left\{\frac{r_\ell+\nu_j^\ell\ \Delta r_\ell}{\Delta r_\ell}\right\}^2 \sum_{n=1}^{NPR} B_{j,n}\, W_{i,n}^{k,\ell\ s} + \left\{\frac{r_\ell+\nu_j^\ell\ \Delta r_\ell}{\Delta r_\ell}\right\} \sum_{n=1}^{NPR} A_{j,n}\, W_{i,n}^{k,\ell,s}$$

$$+ (r_\ell+\nu_j^\ell\ \Delta r_\ell)^2 \qquad (4.11)$$

where ω is an iteration parameter and $(s+\frac{1}{2})$ denotes the velocities at the end of a constant r sweep.

At the element interboundaries, the following conditions are imposed

(i) Continuity of the function:

$$W_{NP\theta,j}^{k,\ell,s+\frac{1}{2}} = W_{i,j}^{k+1,\ell,s+\frac{1}{2}} \qquad \text{for } k=1,\ldots\ldots NE\theta-1 \qquad (4.12)$$

(ii) Continuity of the first derivative:

$$\frac{1}{\Delta\theta_k}\sum_{n=1}^{NP\theta} A_{NP\theta,n}\, W_{n,j}^{k,\ell,s+\frac{1}{2}} - \frac{1}{\Delta\theta_{k+1}}\sum_{n=1}^{NP\theta} A_{i,n}\, W_{n,j}^{k+1,\ell,s+\frac{1}{2}} = 0$$

for $k = 1,\ldots..NE\theta-1$ (4.13)

In addition the following boundary conditions are imposed (Figure 3):

B.C. 1:

$$\frac{1}{\Delta\theta_1}\sum_{n=1}^{NP\theta} A_{i,n}\, W_{n,j}^{1,\ell,s+\frac{1}{2}} = 0 \quad \text{for } \ell=1,\ldots NER \text{ at } \theta = 0 \qquad (4.14)$$

B.C. 2:

(a) $$\frac{1}{\Delta\theta_{NE\theta}}\sum_{n=1}^{NP\theta} A_{NP\theta,n}\, W_{n,j}^{NE\theta,\ell,s+\frac{1}{2}} = 0$$ for $\ell=\ell_f$ and $j=2,\ldots(j_f-1)$

for $\ell<\ell_f$

$\ell = 1,\ldots.\ell_f-1$

$J= 2,\ldots.\ NPR-1$ (4.15a)

(b) $W_{NP\theta,j}^{NE\theta,\ell,s+\frac{1}{2}} = 0$ for $\ell=\ell_f$ $j = j_f,\ldots.NPR-1$

for $\ell>\ell_f$ $\ell=\ell_{f+1},\ldots..NER$

$j = 2,\ldots\ldots..NPR-1$

(4.15b)

Equation (4.11) together with Equations (4.12) to (4.15) were solved line by line at constant r (i.e for $j=2,\ldots.NPR-1$ and $\ell=1,\ldots,NER$)

4.3.2 Constant θ solution:

For constant θ, Equation (4.10) can be written as

$$\omega\, W_{i,j}^{k,\ell,s+1} - \left\{\frac{r_\ell+\nu_j^\ell \Delta r_\ell}{\Delta r_\ell}\right\}^2 \sum_{n=1}^{NPR} B_{j,n}\, W_{i,n}^{k,\ell,s+1}$$

$$- \left\{\frac{r_\ell+\nu_j^\ell \Delta r_\ell}{\Delta r_\ell}\right\} \sum_{n=1}^{NPR} A_{j,n}\, W_{i,n}^{k,\ell,s+1} = \omega\, W_{i,j}^{k,\ell,s+\frac{1}{2}}$$

$$+ \frac{1}{\Delta\theta_k{}^2} \sum_{n=1}^{NP\theta} B_{i,n}\, W_{n,j}^{k,\ell,s+\frac{1}{2}} + (r_\ell+\nu_j^\ell\, \Delta r_\ell)^2 \qquad (4.16)$$

where (s+1) denotes the velocities at end of a constant θ sweep.

At the element interboundaries the following conditions are imposed:

i. Continuity of the function:

$$W_{i,NPR}^{k,\ell,s+1} = W_{i,1}^{k,\ell,s+1} \quad \text{for} \quad \ell=1,\ldots.NER-1 \qquad (4.17)$$

ii. Continuity of the first derivative:

$$\frac{1}{\Delta r_\ell} \sum_{n=1}^{NPR} A_{NPR,n}\, W_{i,n}^{k,\ell,\, s+1} - \frac{1}{\Delta r_{\ell+1}} \sum_{n=1}^{NPR} A_{1,n}\, W_{i,n}^{k,\ell+1,\, s+1} = 0$$

$$\text{for} \quad \ell=1,\ldots.NER-1 \qquad (4.18)$$

In addition, the following boundary conditions are applied,

B.C. 1:

$$\frac{1}{\Delta r_1} \sum_{n=1}^{NPR} A_{1.n}\, W_{i,n}^{k,\ell,s+1} = 0 \quad \text{at } r = 0 \text{ for } k = 1,\ldots NE\theta \qquad (4.19)$$

B.C. 2:

$$W_{i,NPR}^{k,NER,s+1} = 0 \quad \text{at } r = 1 \quad \text{for } k = 1,\ NE\theta \qquad (4.20)$$

Equation (4.16) together with Equations (4.17) to (4.20) were solved line by line at constant θ [i.e. for $i = 2,\ldots NP\theta=1$, and $k = 1,\ldots NE\theta$].

4.4 Computational Scheme:

It has already been stated that the 'Alternating Direction Implicit' ADI method was used to solve the resulting algebraic equations. The system of equations described in Section 4.3.1 was solved line by line at constant r (i.e. for j=2, ...NPR=1 and ℓ=1,...NER). Since the right hand side of the Equation (4.11) is known, the system of equations in Section 4.3.1 can be solved as a one-dimensional problem by the method described in Appendix C. To start the iterative procedure an initial solution was assumed for the entire domain. Due to the nature of the boundary condition along $\theta=\alpha$, two different matrices were obtained. Therefore for part 1, one needs to invert only two matrices regardless of the number of iterations. As discussed in Appendix C, both the left hand side matrices were block diagonal and were converted to a band structure prior to entering the subroutine GELB. One may use LU decomposition equally effectively. After one half iteration the solution is known for j=2, ...NPR-1 but not for J=1 and NPR.The solution at these points may be obtained by smoothing, Chang and Finlayson (1977). However in this work, old values were used at these points for the second half of the iteration scheme and no smoothing was performed. The second half of the iteration scheme (for the system of equations described in Section 4.3.2) is similar to the first half except that only one matrix is inverted regardless of the number of iterations. The computational scheme is shown in Figure 4. After the completion of the second half of the iteration scheme, the velocities at the points marked with solid circles in Figure 5 are still not known. The solution at these points was obtained using a finite difference

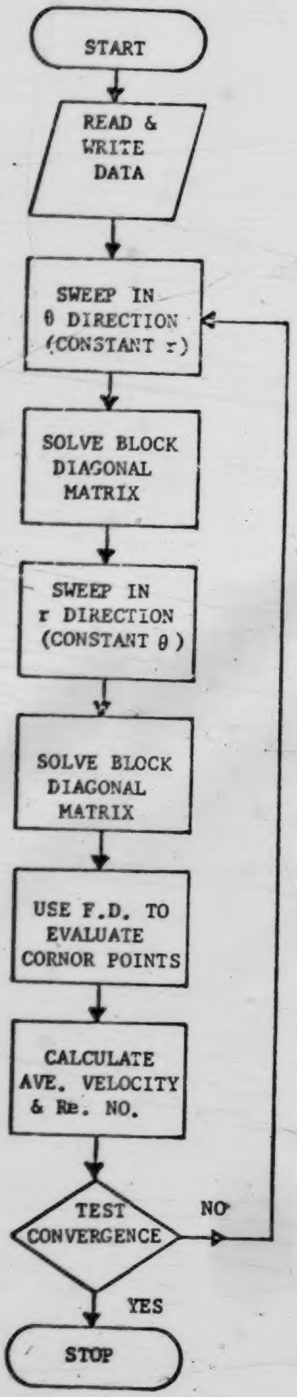
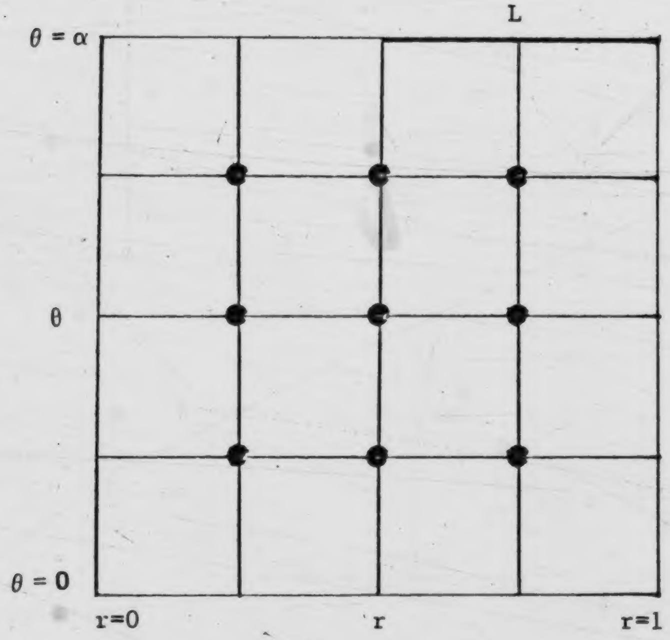

START
READ & WRITE DATA
SWEEP IN θ DIRECTION (CONSTANT r)
SOLVE BLOCK DIAGONAL MATRIX
SWEEP IN r DIRECTION (CONSTANT θ)
SOLVE BLOCK DIAGONAL MATRIX
USE F.D. TO EVALUATE CORNOR POINTS
CALCULATE AVE. VELOCITY & Re. NO.
TEST CONVERGENCE
NO
YES
STOP

FIGURE 4. Flow Chart for the Computational Scheme for Finned Tube



● POINTS WHERE FINITE DIFFERENCE METHOD WAS APPLIED

FIGURE 5. Location of the Points for Finite Difference Method

technique. The finite difference development of Equation (4.7) is given in Appendix D.

Once the solution at all the points was known, the procedure was repeated till convergence was achieved. Convergence was assumed to have been reached when the average velocity did not change by more than 10^{-4} in 50 iterations. A quadrature approach was used to calculate the average flow velocity.

The rate of convergence was found to be a very strong function of the iteration factor ω. The iterative procedure becomes unstable when ω is large and the rate of convergence is very slow when ω is small. The optimal or near optimal value of ω was found by trial and error. In general, the appropriate value of ω increased with the number of interior collocation points and with the number of elements. Table 1 shows that ω varied from 2 (for $NE\theta = NER = 5$, $NP\theta = NPR = 3$) to 1150 (for $NE\theta = NER = 2$, $NP\theta$, $NPR = 8$). The optimal value of ω was also a function of the number of fins and the fin length.

As the iteration parameter is the recriprocal of a time interval when solving similar problems with the time derivative of W included, a large ω means that the time step is small and the number of the time intervals to reach "steady state" is large. Table 1 shows that as ω is increased the number of iterations to convergence also increased. The CPU time per 100 iterations is shown in Table 1. The computational time was found to increase fairly rapidly with the number of collocation points. For example, for two elements in the θ and r directions, the CPU time per 100 iterations was 1.7 and 22.4

seconds for $NP\theta$ = NPR = 5 and $NP\theta$ = NPR = 10, respectively. For this type of problem, the ADI method becomes fairly expensive in terms of CPU demand for cases using a large number of collocation points.

4.5 Calculation of the Average Velocity:

The average velocity in finned tubes is obtained using the following expression:

$$< W > = \frac{\int_{\theta=0}^{\alpha} \int_{r=0}^{1} W \, r \, dr \, d\theta}{\int_{\theta=0}^{\alpha} \int_{r=0}^{1} r \, d \, r \, d\theta} \qquad (4.21)$$

or

$$< W > = \frac{2}{\alpha} \int_{0}^{\alpha} \int_{0}^{1} W \, r \, dr \, d\theta \qquad (4.22)$$

Equation (22) was evaluated using a quadrature approach. The following formula was used to evaluate the average velocity

$$< W > = \sum_{k=1}^{NE\theta} \sum_{i=1}^{NP\theta} w_i \sum_{\ell=1}^{NER} \sum_{j=1}^{NPR} (w_j \, W_{i,j}^{k,\ell} \, r_j) \qquad (4.23)$$

The listing of the weighting factor w is given in Appendix B. Integration was performed first in the r-direction. The resulting values were integrated again in the θ-direction to complete the integration over the entire flow region of interest. The velocities at the element interboundaries do not affect the average velocity as the quadrature at these points is zero.

Table 1

Summary of Computations for the
Orthogonal Collocation on Finite Elements Method

L	NF	NEθ NER	NPθ NPR	ω	CPU (seconds)	Total No. of Iterations
0.3	3	2	5	40	1.7	250
0.3	3	2	6	70	2.2	250
0.3	3	2	7	120	6.3	750
0.3	3	2	8	185	10	1100
0.5	3	2	5	40	1.7	250
0.5	3	2	6	70	2.2	300
0.5	3	2	7	110	6.3	600
0.5	3	2	8	170	10	500
0.7	3	2	5	40	1.7	300
0.7	3	2	6	80	2.2	500
0.7	3	2	7	120	6.3	800
0.7	3	2	8	180	10	750
0.7	3	2	10	385	22.4	1600
0.4	8	2	5	330	1.7	250
0.4	8	2	6	500	2.2	600
0.4	8	2	7	780	6.3	
0.4	8	2	8	1150	10	1000
0.3	3	5	3	2	1.7	100
0.3	3	5	5	250	6.1	1000
0.5	3	5	3	2	1.7	100
0.5	3	5	5	300	6.1	250
0.7	3	5	3	2	1.7	100
0.7	3	5	5	300	6.1	200

One quantity which is quite useful in the study of fluid flow in finned tubes is the product of friction factor and Reynolds number (f.Re). It is given by the following expression, Nandakumar and Masliyah (1975).

$$f.Re = \frac{8A_F^2}{\langle W\rangle \; C_F^2} \qquad (4.24)$$

where $\langle W\rangle$ is the average velocity, A_F is the cross-sectional area, and C_F is the wetted perimeter. Here Re is based on the equivalent diameter D_e.

The representative flow area and the wetted perimeter are given by, respectively,

$$A_F' = \pi R^2\left(\frac{\alpha}{2\pi}\right) = \frac{\alpha}{2} R^2 \qquad (4.25)$$

and

$$C_F' = (2\pi R)\left(\frac{\alpha}{2\pi}\right) + L' = \alpha R + L' \qquad (4.26)$$

where

$$A_F = A_F'/R^2 \qquad (4.27)$$

$$C_F = C_F'/R \qquad (4.28)$$

and

$$L = L'/R \qquad (4.29)$$

Using Equations (4.25) to (4.28) Equation (4.24) becomes

$$f.Re = \frac{2\alpha 2}{\langle W\rangle \; (\alpha + L'/R)^2} \qquad (4.30)$$

In a case when there is no fin present (L=0), Equation (4.30) becomes

$$f.Re = \frac{2}{\langle W\rangle} \qquad (4.31)$$

Equation (4.31) is used to check the numerical results presented in the Section 4.7

4.6 Other Techniques

4.6.1 Least Square Matching Technique

The general solution of the Poisson equation representing the momentum equation is well known and is given by

$$W = b_o + \frac{r^2}{4} + b_1 \ln r + \sum_k (a_k r^{-k} + b_k r^k) \cos \theta k$$

$$+ \sum_k (c_k r^{-k} + d_k r^k) \sin \theta k$$

Utilizing the boundary conditions (4.3c) and the fact that the solution must be finite at r=0, a more specific solution is,

$$W = \frac{r^2}{4} + \sum_{k=0}^{M} a_k r^k \cos \theta k$$

The coefficients a_k can be determined by choosing N(=M+1) points along the flow duct boundaries. Each boundary collocation point provides one algebraic equation. The resulting N simultaneous equations can be solved for the N coefficients. However, by considering more boundary collocation points ($M \sim \frac{1}{3} N$) than coefficients, the over-determined set of algebraic equations can be reduced to a set of (M+1) equations by a least square approach with a weighting factor of unity.

Although this method was found to be very successful in the solution of flow in arbitrarily shaped ducts, Ratkowsky and Epstein (1968) and other complicated flows, Bowen and Masliyah (1973), the method failed to give any meaningful flow field for fin lengths greater than 0.2. The number of coefficients varied between 5 and 20. Similar conclusions were also reached by Soliman and Feingold (1977).

4.6.2 Finite Difference Method, F.D.

Masliyah (1975) has used a F.D. method to solve Equation (4.2). The momentum equation was discretized using a three-point central difference module. The derivatives at the flow boundaries were approximated by Newton forward and backward three-point formulae. A successive over-relaxation method was used with a relaxation factor of 1.7. Solutions for grids of (11x11), (21x21) and (41x41) were obtained.

Convergence for the (11x11) grid was fast and the rate of convergence was found to decrease rapidly as the number of the grid points was increased. For a grid of (41x41), the rate of convergence was so slow for the case of a fin length, L = 0.4 and number of fins NF = 8 that it was not possible to ascertain whether convergence had occurred after a total of 7000 iterations.

Table 2 shows the time requirements and the total number of iterations needed to achieve convergence. In general, convergence was assumed to have been reached when the average velocity did not change by more than 10^{-5} in 50 iterations.

4.6.3 Soliman and Feingold Approach.

Due to the failure of the least square matching approach, and to overcome the mixed-type boundary conditions, the flow domain was divided into two regions separated by a circular arc of radius (1-L), Soliman and Feingold (1977). General trial functions for each region were evaluated. These functions satisfied the respective region boundary conditions. Using the continuity of velocity and its derivative at the boundary of the two regions at equi-distant collocation points, the constants contained in the trial flow functions were

Table 2

Summary of Computations for the Finite Difference Method

		CPU time in seconds per 100 iterations			Total number of iterations to convergence		
NF	L	11x11	21x21	41x41	11x11	21x21	41x41
3	0.3	0.12	0.46	2.0	1000	3700	7000
3	0.5	0.12	0.46	2.0	800	2400	2500
3	0.7	0.12	0.46	2.0	700	3400	2600
8	0.4	0.12	0.46	2.0	800	2500	(7000) N/C

N/C no convergence,

evaluated. The number of inter-boundary collocation points varied between 10 and 20. Soliman and Feingold found that the average velocity of the flow was within 1% for 10 and 20 coefficients. The results using 20 coefficients are given in Table 3.

4.7 Discussion of Results

In order to compare the results for the fluid flow in an internally finned tube, the central velocity and the average velocity will be used for comparison. In order to gain confidence in the numerical results, a limiting case is considered. When the fin length is zero the exact value of f.Re is 16. OCFE also gives a value of 16. This shows that the numerical results are in perfect agreement with the exact solution for the limiting case considered.

As the purpose of this work is not to study the flow in finned tubes, but rather to study the general applicability of OCFE to obtain solutions to this type of problem, only a few flow cases are considered, and these cases are primarily dictated by the availability of results from other workers. The flow cases considered are for NF = 3, 8 with fin lengths of 0.3, 0.4, 0.5 and 0.7. NF is the number of fins.

H. Kan (1978) has presented some results for GOC. Due to the presence of a discontinuity along $\theta = \alpha$(presence of the fin), global orthogonal collocation is of limited application. It is not possible to arbitrarily select a fin length, since the tip of the fin must lie on a collocation point. When NR is an odd number, the middle collocation point is always at r = 0.5. It is for this reason that the results for GOC are only given for one fin length,

Table 3

Summary of Results for the OCFE and FD

NF = 3

L		1	2	3	4	5	6	7	8	9	10	11
		NER=2 NPR=5	NER=2 NPR=6	NER=2 NPR=7	NER=2 NPR=8	NER=2 NPR=10	NER=5 NPR=3	NER=5 NPR=5	FD 11x11	FD 21x21	FD 41x41	S&F
0.3	W_c	0.2103	0.2124	0.2136	0.2143	-	0.204	0.209	0.2131	0.2142	0.2140	0.1959
	<W>	0.09541	0.09689	0.09761	0.09800	-	0.09124	0.0946	0.09657	0.09791	0.09813	0.09464
0.5	W_c	0.143	0.1492	0.1522	0.1542	-	0.139	0.1518	0.153	0.1553	0.1560	0.1429
	<W>	0.06197	0.06457	0.06582	0.06662	-	0.0603	0.06473	0.06538	0.06705	0.06762	0.06555
0.7	W_c	0.0642	0.0726	0.07716	0.07974	0.0825	0.067	0.07737	0.079	0.0822	0.0832	0.08024
	<W>	0.03987	0.04158	0.04255	0.04313	0.04375	0.0398	0.04237	0.04246	0.04362	0.04403	0.04388
			NF = 8									
0.4	W_c	0.122	0.127	0.1304	0.1320				0.1272	0.1319	N/C	0.1305
	<W>	0.0395	0.0421	0.04349	0.04424				0.04175	0.04399	N/C	0.04491

S&F - Soliman and Feingold, Private Communication with Dr. H.M. Hassan

N/C - no convergence

namely L = 0.5. Cases for NR=Nθ=5,7 and 9 were attempted using the global orthogonal collocation method.

For the case of orthogonal collocation on finite element, two configurations were attempted.

(a) NER = NEθ = 5, NPR = NPθ = 3 and 5 with Δr and Δθ being equal for each element. Use of an odd number of collocation points ensured that the fin tip lies on a collocation point (See Figure 3a).

(b) NER = NEθ = 2 with NPR = NPθ = 5,6,7,8 and 10. Here Δr was not equal for the two elements and therefore Δr_1 was selected so that the fin tip falls on a collocation point. (See Figure 3b).

The summary of results is given in Table 3. For comparison, the finite difference solution for the fine mesh (41x41) will be considered as the "true" solution. Indeed, a close examination of Table 3, columns 9 and 10, indicates that both the centre velocity W_c and the average velocity, <W> for the (21x21) grid and the (41x41) grid are in fair agreement (maximum variation is about 1.2%).

The results for global orthogonal collocation for a fin length of 0.5 lie much lower than the "true" solution. The values of W_c and W are shown in Figure 6. The fluid problem employed here is a rather severe test of the GOC method. For NR = 9, the values of r at the 4th, 5th, and 6th collocation points are, 0.2971, 0.5000 and 0.7029, respectively. This means that the first collocation point away from the fin tip, where the velocity is not zero and the symmetry condition is applied, is at a distance of 0.2029 away from the fin tip.

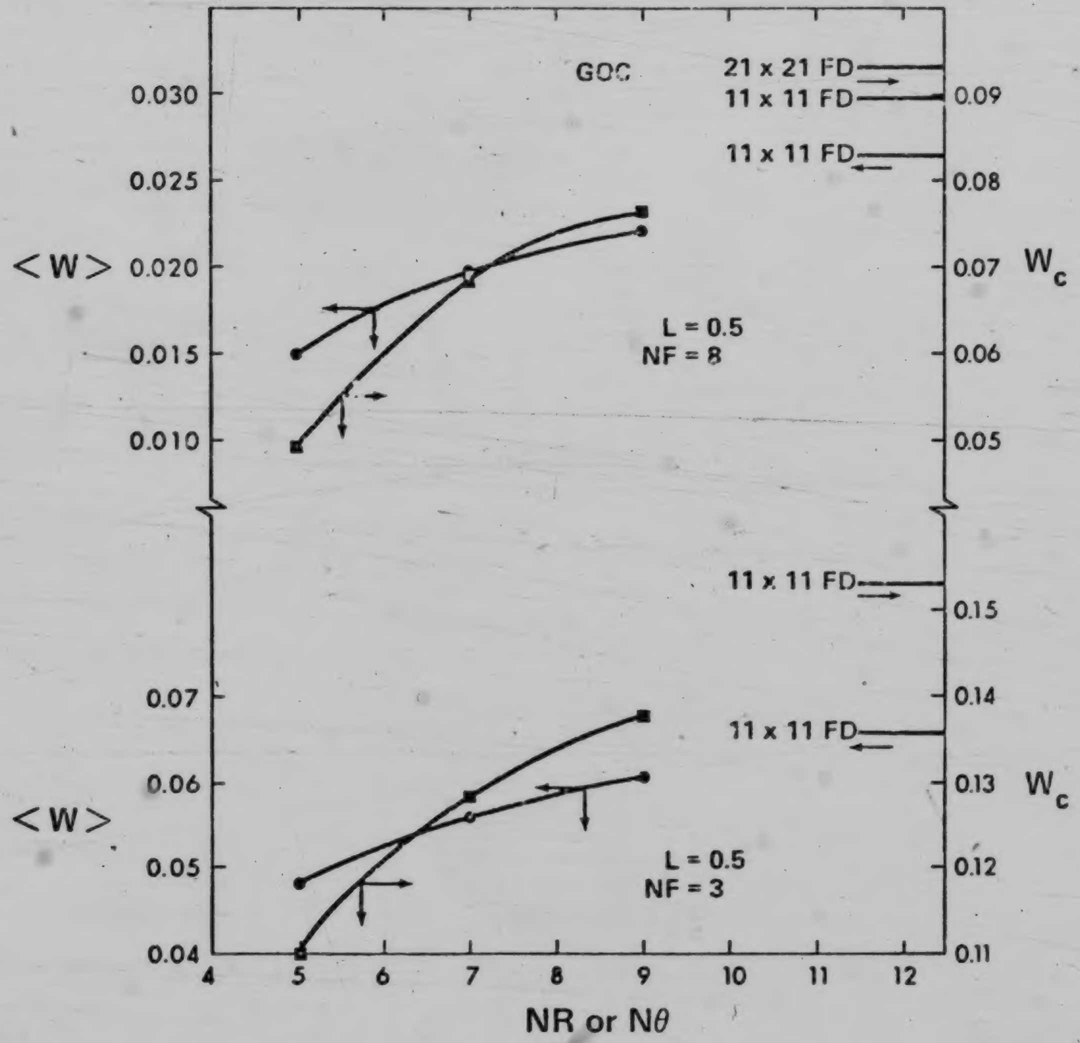


FIGURE 6. Variation of Centre and Average Velocities for L=0.5 with Number of Collocation Points for GOC

In other words, the gap between the 4th and the 5th collocation points is fairly large and therefore the velocity resolution is fairly poor near the fin tip. As the fin tip position is very critical in determining the velocity field, it is not surprising that the results as given by the GOC are poor, although a total of 49 interior collocation points are used.

The results for the OCFE for the case of NPθ = NPR = 3 and 5 with NEθ = NER = 5 are given in columns 6 and 7 of Table 3. As the total number of collocation points is increased from 3 to 5, the agreement with the "true" solution improved. The maximum difference is about 7% (for the case of W_c, L = 0.7). For NPR = 5, the collocation points are 0, 0.1127, 0.5, 0.8873 and 1.0. With $\Delta r = 0.2$, this means that the interval between the second collocation point and the third collocation point (j_f) is 0.2 (0.5-0.1127)=0.07746. A finite difference method with a 14x14 grid would produce a uniform Δr similar to that near the fin tip for the case of NEθ = NER = 5 and NPθ = NPR = 5. Comparison of column 7 with columns 8 and 9 of Table 3 shows that the results of OCFE do not fall between those given by the finite difference method for the (11x11) and the (21x21) grids. In fact the OCFE results fall below those for the (11x11) grid. This indicates that the OCFE having uniform element size with NEθ = NER = 5 and NPθ = NPR = 5 (a total of 25x9 collocation points) is not suitable for this type of a problem.

Further numerical experimentation was conducted with two elements of unequal size in r-direction and two elements of equal size in θ-direction, i.e., NEθ = NER = 2. The number of collocation points was varied, viz, $NP\theta = NPR = 5,6,7,8$ and 10. As the number of collocation points was increased, the values of W_c and $\langle W \rangle$ approached those given by the finite difference method with a (41x41) grid. Figures 7 and 8 show the variation of the centre velocity and the average velocity with the number of collocation points, respectively. The values of W_c and $\langle W \rangle$ for the $NF = 3$ and $L = 0.3$ for the case of $NP\theta = NPR = 8$ (total of 144 interior collocation points) are close to those given by a grid of about 21x21 using the finite difference method. Similarly, using W_c and $\langle W \rangle$ as basis for comparison, for $NF = 3$ and $L = 0.5$ and 0.7, the $NP\theta = NPR = 8$ case was found to be equivalent to a grid of about (15x15). For the case of a more number of fins, $NF = 8$, a $NP\theta = NPR = 6$ (total of 64 interior collocation points) was found to be equivalent to the finite difference scheme of (11x11) grid and a $NP\theta = NPR = 8$ was found to be equivalent to at least a (21x21) grid.

The results of Soliman and Feingold are shown in Figures 7 and 8 for comparison. For the case of $L = 0.7$ ($NF=3$) and $L = 0.4$ ($NF=8$) their results are in good agreement with those for $NP\theta = NPR = 8$. However, as the fin length is decreased, Soliman and Feingold results become equivalent to those of lower order grid points.

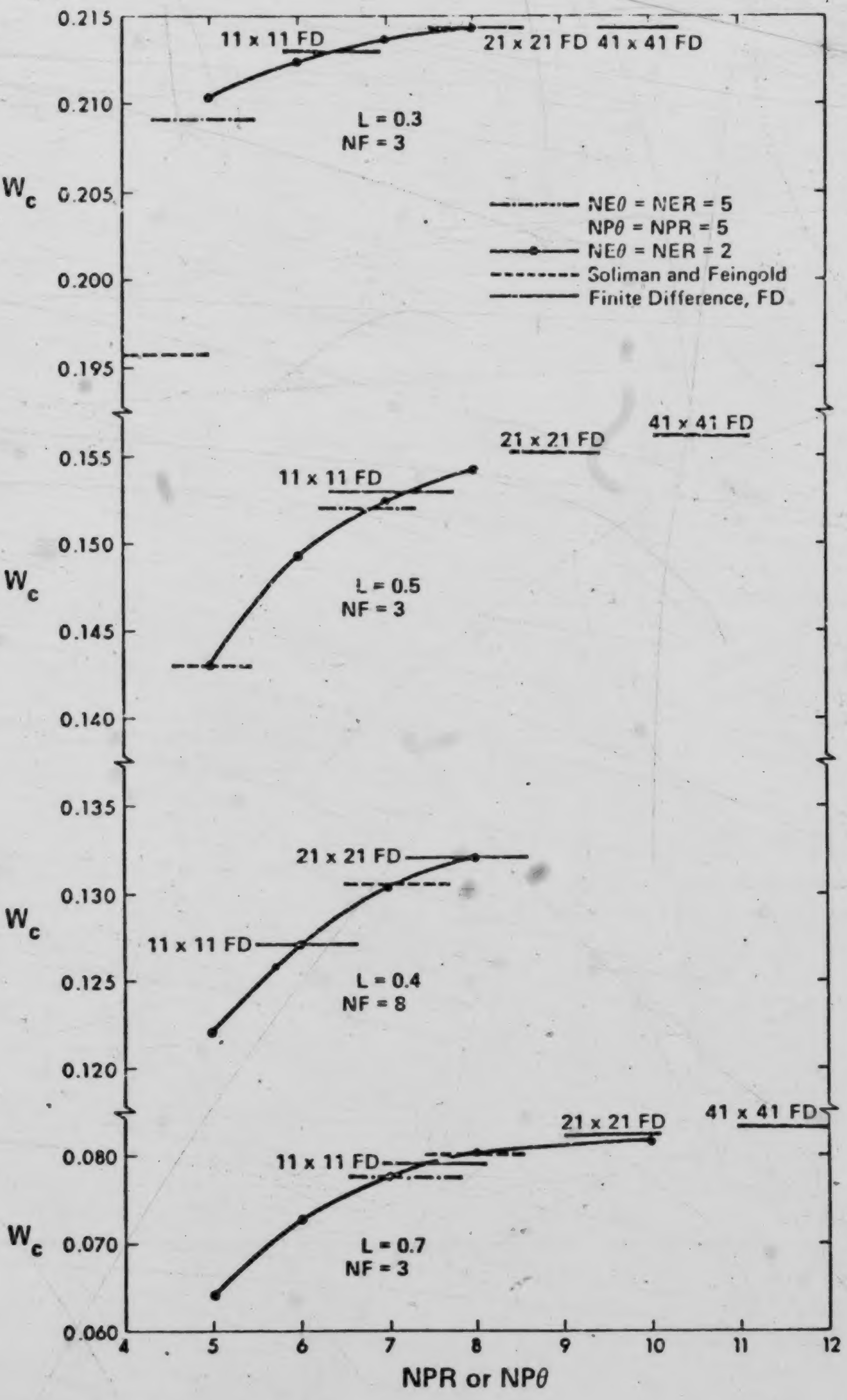


FIGURE 7. Variation of Centre Velocity with Number of Collocation Points for OCFE

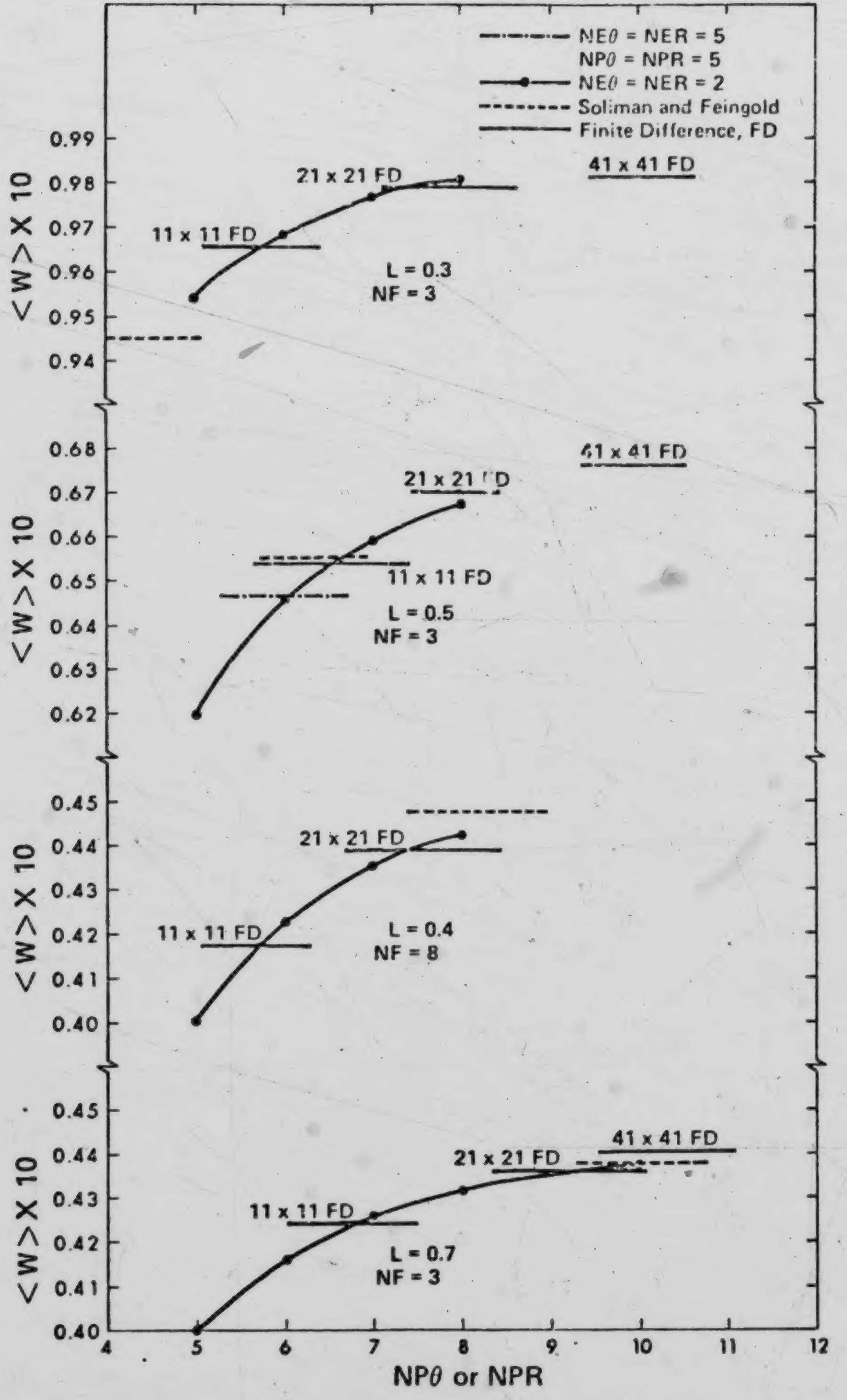


FIGURE 8. Variation of Average Velocity with Number of Collocation Points for OCFE

CHAPTER 5

APPLICATION OF THE METHOD OF ORTHOGONAL COLLOCATION ON FINITE ELEMENTS TO POROUS MEDIA

This chapter demonstrates the applicability of the method of orthogonal collocation on finite elements (OCFE) to a flow problem in porous media. The equations simulating miscible displacement in porous media were solved using OCFE. Unlike the finned tube problem, a direct method of solution was used to solve the resulting algebraic equations.

The physical process considered here is that of a homogenous porous medium having a very viscous and practically immobile oil. A solvent of low viscosity is injected into the reservoir through an injection well in order to reduce the viscosity of the oil. The oil is assumed to be highly miscible with the solvent and the oil-solvent mixture viscosity is taken to be a strong function of the solvent concentration. As solvent injection proceeds, a mixture of oil and solvent is produced through the production well.

5.1 Governing Equations:

The two dimensional incompressible miscible displacement in a porous media can be described by the following equations, Settari et al. (1976).

$$- \underline{\nabla} \cdot \underline{u} = q \qquad (5.1)$$

and

$$\underline{\nabla} \cdot (\phi \, K_{DT} \, \underline{\nabla} \, C) - \underline{\nabla} \cdot (\underline{u} \, C) = \phi \frac{\partial C}{\partial t} + q \, C_{in} \qquad (5.2)$$

where ϕ is the porosity of the porous medium, $\underline{u}$ is the Darcy velocity and K_{DT} is the total dispersion coefficent. C_{in} is the source concentration and C is the concentration of the solvent in the oil-solvent mixture. q is the amount of fluid injected or produced in cubic centimeters per cubic centimeter of the formation per second.

The viscosity of the oil-solvent mixture is assumed to be a function of concentration, and is given by, Settari et al. (1976).

$$\mu = \frac{\mu_o \mu_s}{[(1-C)\mu_s{}^{1/e} + C\ \mu_o{}^{1/e}]^e} \tag{5.3}$$

where μ_o and μ_s are the viscosities of the oil and the solvent, respectively, and e is a mixing parameter.

The two-dimensional form of Equations (5.1) and (5.2) can be written, respectively, as

$$-\frac{\partial}{\partial x'} u_x - \frac{\partial}{\partial y'} u_y = q \tag{5.4}$$

and

$$\frac{\partial}{\partial x'} \left\{ K_D \frac{\partial C}{\partial x'} - u_x C \right\} + \frac{\partial}{\partial y'} \left\{ K_D \frac{\partial C}{\partial y'} - u_y C \right\} = \phi \frac{\partial C}{\partial t} + q(x', y')\ C_{in} \tag{5.5}$$

where $K_D = \phi\ K_{DT}$

Using Darcy's law the components of the fluid velocity in Equation (5.4) can be replaced by

$$u_x = -\frac{K_p}{\mu} \frac{\partial p}{\partial x'} \tag{5.6}$$

and

$$u_y = -\frac{K_p}{\mu} \frac{\partial p}{\partial y'} \tag{5.7}$$

where K_p is the permeability of the porous medium. The dispersion coefficient K_D is taken as a constant in both directions and is assumed to be independent of the fluid velocity.

Multiplying equation (5.4) by C and using equation (5.5) yields

$$\frac{\partial}{\partial x'}\left[K_D \frac{\partial C}{\partial x'}\right] + \frac{\partial}{\partial y'}\left[K_D \frac{\partial C}{\partial y'}\right] - u_x \frac{\partial C}{\partial x'} - u_y \frac{\partial C}{\partial y'} = \phi \frac{\partial C}{\partial t} + q(x', y')\,(C_{in}-C) \qquad (5.8)$$

where C_{in} is the source concentration. This concentration is equal to the input concentration for an injection well and to C(x, y) for a production well. Consequently the last term in Equation (5.8) disappears for all production wells.

The boundary conditions considered are for an isolated system. The geometry of the porous medium is shown in Figure 9. The porous medium is assumed to be a rectangle. Equations (5.4) and (5.8) were solved using the following boundary conditions,

$$\frac{\partial P}{\partial n} = 0 \qquad \text{for } (x,y) \in d\Lambda \qquad (5.9a)$$

$$\frac{\partial C}{\partial n} = 0 \qquad \text{for } (x,y) \in d\Lambda \qquad (5.9b)$$

$$u_x = u_y = 0 \qquad \text{for } (x,y) \in d\Lambda \qquad (5.9c)$$

$$C(x,y,0) = 0 \qquad \text{for } (x,y) \in d\Lambda \qquad (5.9d)$$

where $d\Lambda$ is the boundary of Λ and n is the outward normal to $d\Lambda$.

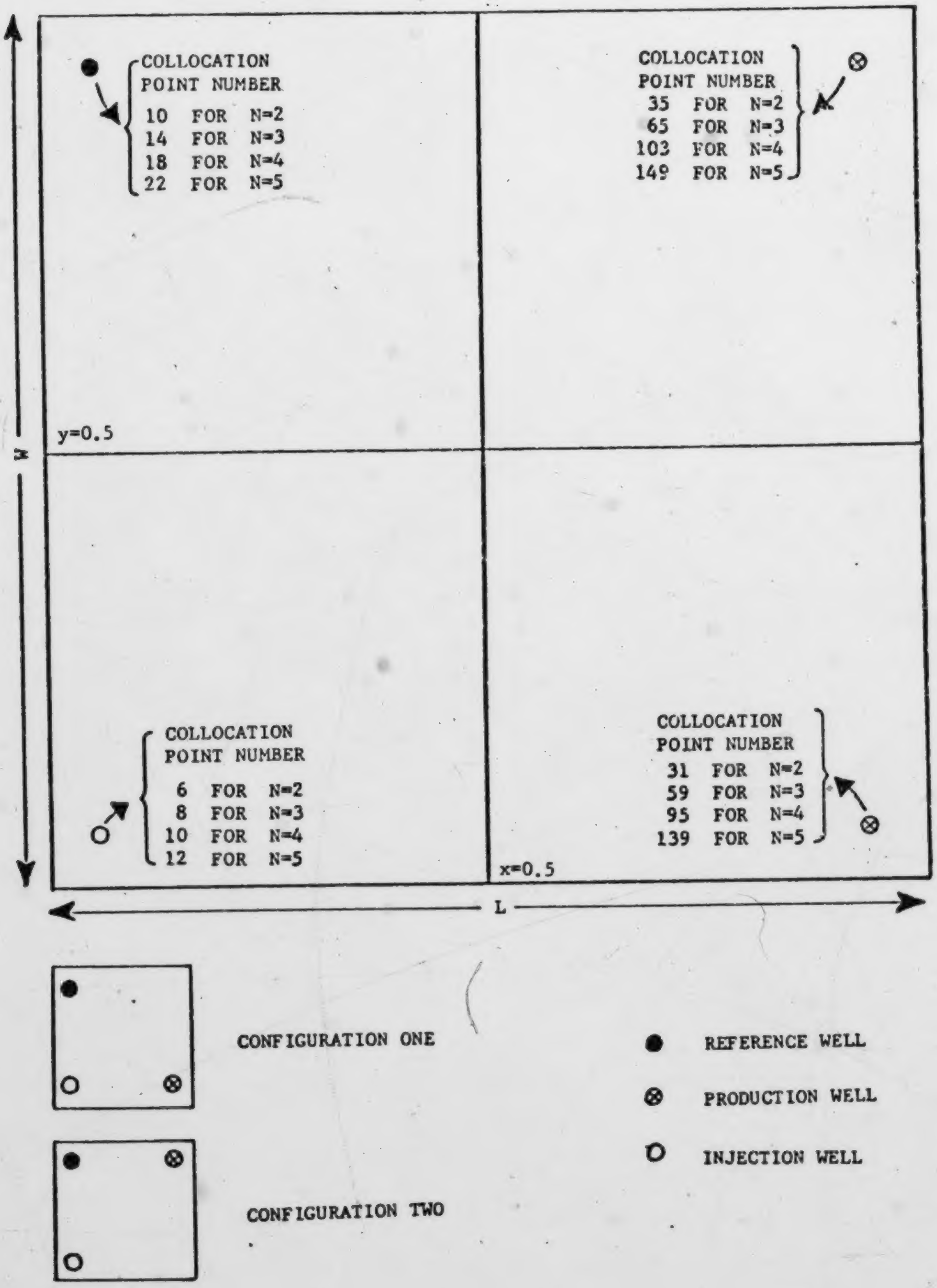


FIGURE 9 Geometry of the Porous Medium

5.2 Numbering Scheme

For computational purposes a quarter of a five-spot was considered. Only two elements of equal size were used in each direction. The number of interior collocation points was varied from 2 to 5. Five interior collocation points correspond to a total of 160 unknowns. The numbering scheme for N=2,3,4 and 5 is shown in Figures 10 to 13, respectively. The corner points of the blocks were not assigned any number,since they did not appear in OCFE formulation of the equations.

As shown in Figures 10 to 13, the injection well can always be identified as being the first interior collocation point in the increasing direction of both x and y. Two different locations of the production well were considered. In one scheme the production well was located diagonally opposite to the injection well. In the other scheme, the production well was such that the injection and the production wells were symmetric to the line x = 0.5. (In this situation the geometry does not represent a five-spot). At any time, only one production and one injection well were considered, however, the computer program can handle any number of production and injection wells.

5.3 OCFE Formulation of the Governing Equations:

5.3.1 Continuity Equation:

Considering Equation (5.4) and using Darcy's law, one obtains

$$\frac{\partial}{\partial x'} \left[\frac{K_p}{\mu} \frac{\partial p}{\partial x'}\right] + \frac{\partial}{\partial y'} \left[\frac{K_p}{\mu} \frac{\partial p}{\partial y'}\right] = q\ (x',y') \tag{5.10}$$

For a porous medium of constant permeability, Equation (5.10) becomes

$$\frac{\partial}{\partial x'} \left[\frac{1}{\mu} \frac{\partial p}{\partial x'}\right] + \frac{\partial}{\partial y'} \left[\frac{1}{\mu} \frac{\partial p}{\partial y'}\right] = \frac{q}{K_p \times 14.696764} \tag{5.11}$$

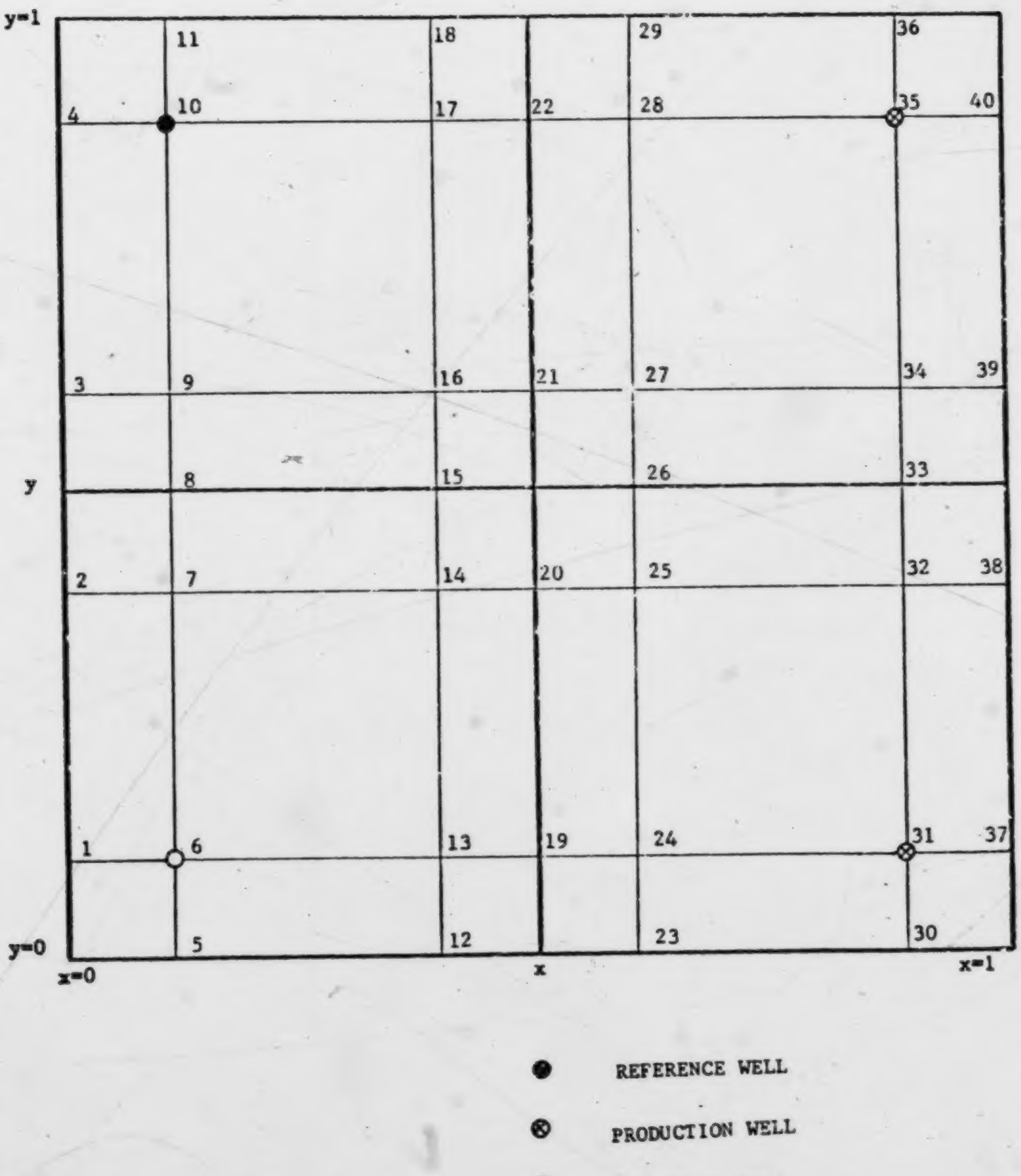


FIGURE 10. Numbering of Unknowns for N=2

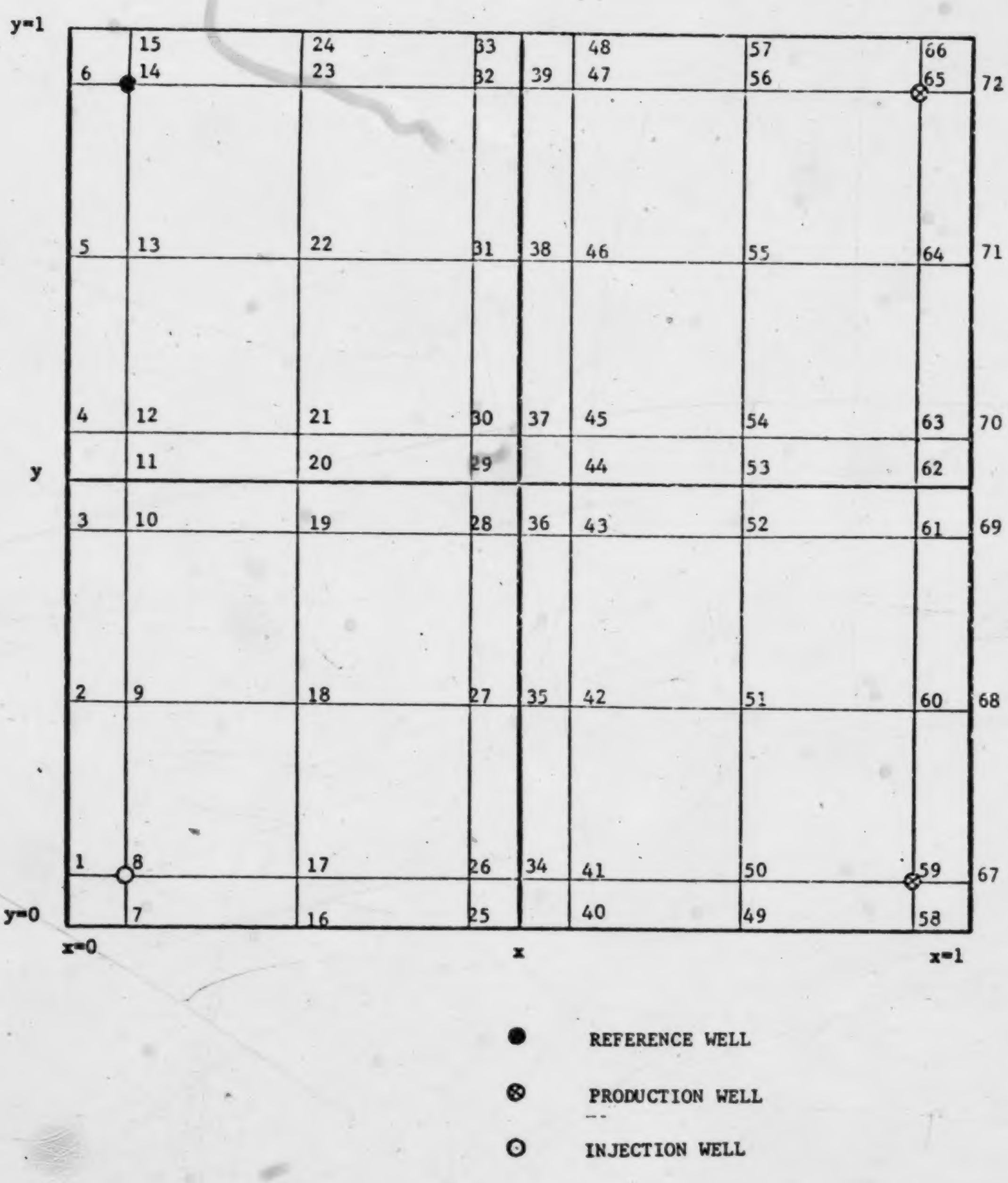


FIGURE 11. Numbering of Unknowns for N=3

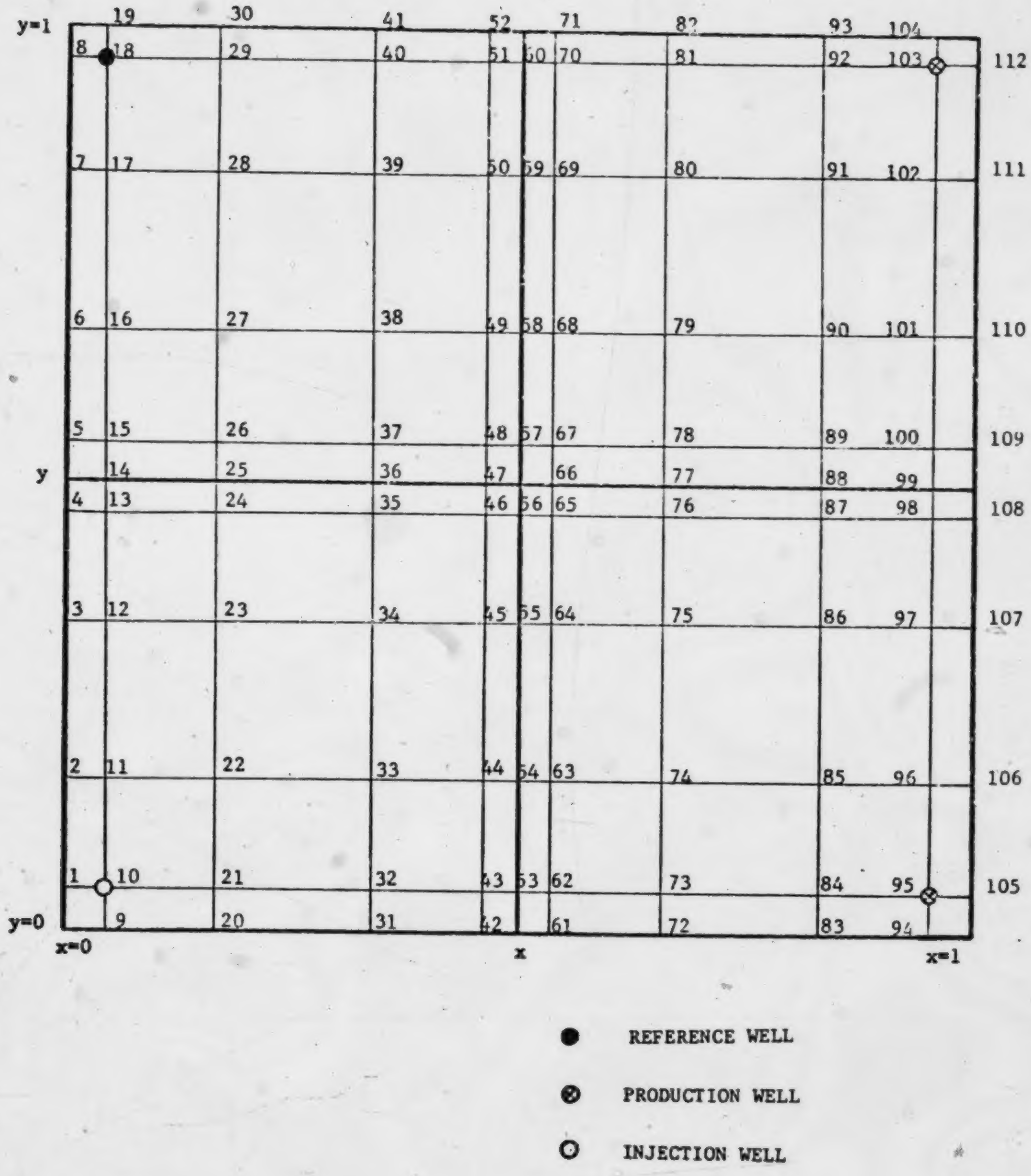


FIGURE 12. Numbering of Unknowns for N=4

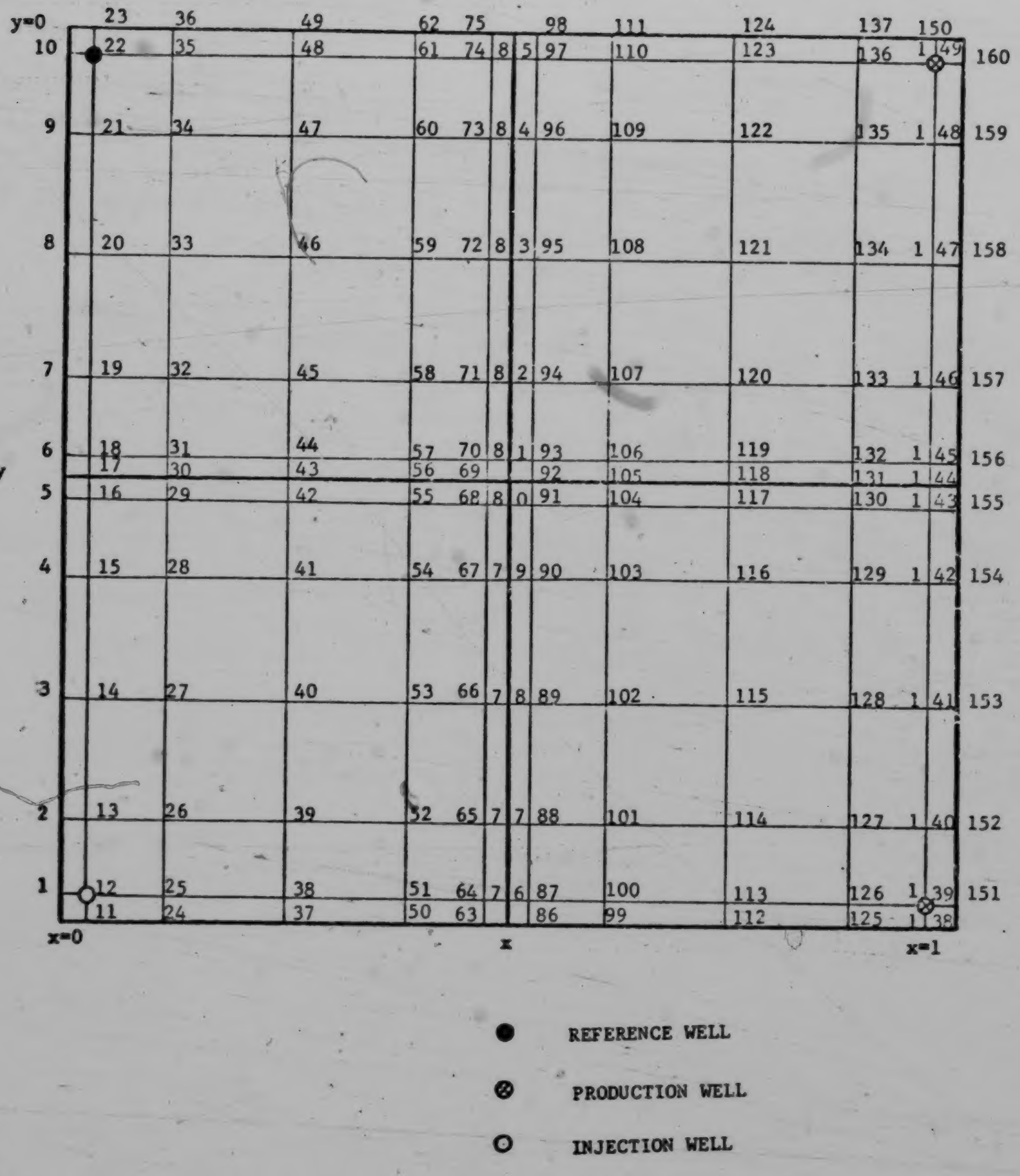


FIGURE 13. Numbering of Unknowns for N=5

The constant 14.696764 is for dimensional consistency

$$\text{Let } x' = xL \tag{5.12}$$

$$\text{and } y' = yW \tag{5.13}$$

Here L and W are the length and the width of the formation under consideration and x and y are the dimensionless coordinates.

Substituting the dimensionless quantities in Equation (5.10) and rearranging, one obtains

$$\mu \frac{\partial^2 p}{\partial x^2} - \frac{\partial \mu}{\partial x} \frac{\partial p}{\partial x} + Z^2 \left[\mu \frac{\partial^2 p}{\partial y^2} - \frac{\partial \mu}{\partial y} \frac{\partial P}{\partial y}\right] = \frac{q\, L^2 \mu^2}{K_p * 14.696764} \tag{5.14}$$

$$\text{where } Z = L/W \tag{5.15}$$

In Equation (5.14) only x, y and Z are dimensionless.

Two new variables g and ν are defined such that

$$g = \frac{x - x_k}{\Delta x_k} \tag{5.16a}$$

and

$$\nu = \frac{y - y_\ell}{\Delta y_\ell} \tag{5.16b}$$

where $\Delta x_k = x_{k+1} - x_k$

and $\Delta y_\ell = y_{\ell+1} - y_\ell$

Applying the method of orthogonal collocation on finite elements to Equation (5.1), and using the new independent variables g and ν as defined by Equation (5.16), one obtains,

$$\frac{\mu}{\Delta x_k^2} \sum_{n=1}^{NPX} B_{i,n} P^{k,\ell}_{n,j} - \frac{1}{\Delta x_k} \left\{ \sum_{n=1}^{NPX} A_{i,n} \mu^{k,\ell}_{n,j} \right\} \left\{ \frac{1}{\Delta x_k} \sum_{n=1}^{NPX} A_{i,n} P^{k,\ell}_{n,j} \right\}$$

$$+ Z^2 \left[\frac{\mu}{\Delta y_\ell^{\,2}} \sum_{n=1}^{NPY} B_{j,n} P^{k,\ell}_{i,n} - \frac{1}{\Delta y_\ell^{\,2}} \left\{ \sum_{n=1}^{NPY} A_{j,n} \mu^{k,\ell}_{i,n} \right\} \left\{ \sum_{n=1}^{NPY} A_{j,n} P^{k,\ell}_{i,n} \right\} \right]$$

$$= \frac{q(x_i,y_j)\, L^2\, \mu^2}{K_p * 14.696764} \qquad (5.17)$$

where

$$\begin{aligned} i &= 2,\ldots\ldots,NPX-1 \\ j &= 2,\ldots\ldots,NPY-1 \\ k &= 1,\ldots\ldots,NEX \\ \ell &= 1,\ldots\ldots,NEY \end{aligned} \qquad (5.18)$$

NEX and NEY are the number of elements in x and y directions, respectively. NPX and NPY are the total number of collocation points (including boundary points) in any element $k\ell$, in the x and y directions, respectively.

Multiplying Equation (5.17) by Δx_k^2 and letting $Z \frac{\Delta x_k}{\Delta y_\ell} = F$, yields,

$$\mu \sum_{n=1}^{NPX} B_{i,n} P^{k,\ell}_{n,j} - \left\{ \sum_{n=1}^{NPX} A_{i,n} \mu^{k,\ell}_{n,j} \right\} \left\{ \sum_{n=1}^{NPX} A_{i,n} P^{k,\ell}_{n,j} \right\}$$

$$+ F^2 \left[\mu \sum_{n=1}^{NPY} B_{j,n} P^{k,\ell}_{i,n} - \left\{ \sum_{n=1}^{NPY} A_{j,n} \mu^{k,\ell}_{i,n} \right\} \left\{ \sum_{n=1}^{NPY} A_{j,n} P^{k,\ell}_{i,n} \right\} \right]$$

$$= \frac{q(x_i,y_j)\, L^2\, \mu^2\, \Delta x_k^{\,2}}{K_p * 14.696764} \qquad (5.19)$$

where i, j, k, and ℓ vary according to Equation (5.18). The following boundary conditions apply:

$$\sum_{n=1}^{NPX} A_{1,n} \, P_{n,j}^{1,\ell} = 0$$

For $\ell=1, \ldots NEY$

and $j=2, \ldots . NPY-1$ (5.20)

$$\sum_{n=1}^{NPX} A_{NPX,n} \, P_{n,j}^{NEX,\ell} = 0$$

and

$$\sum_{n=1}^{NPY} A_{1,n} \, P_{i,n}^{k,\ell} = 0$$

For $k=1, \ldots . NEX$

$i=2, \ldots . NPX-1$ (5.21)

$$\sum_{n=1}^{NPY} A_{NPY,n} \, P_{i,n}^{k,NEY} = 0$$

At the element interboundaries the first derivative is assumed to be continuous leading to

$$\frac{1}{\Delta x_k} \sum_{n=1}^{NPX} A_{NPX,n} \, P_{n,j}^{k,\ell} - \frac{1}{\Delta x_{k+1}} \sum_{n=1}^{NPX} A_{1,n} \, P_{n,j}^{k+1,\ell} = 0$$

for $\ell=1, \ldots . NEY$

$j=2, \ldots . NPY-1$ (5.22A)

and $k=1$

and

$$\frac{1}{\Delta y_\ell} \sum_{n=1}^{NPY} A_{NPY,n} \, P_{i,n}^{k,\ell} = \frac{1}{\Delta y_{\ell+1}} \sum_{n=1}^{NPY} A_{1,n} \, P_{i,n}^{k,\ell+1} \qquad (5.22B)$$

for $k=1, \ldots NEX$

$i=2, \ldots NPX-1$

and $\ell=1$

A simultaneous solution of the system of equations as described by Equations (5.19) to (5.22) was obtained by a direct method using the LU decomposition technique.

5.3.2. Convection-Diffusion Equation

Assuming a constant dispersion coefficient, Equation (5.8) can be wrriten as

$$K_D \frac{\partial^2 C}{\partial x'^2} + K_D \frac{\partial^2 C}{\partial y'^2} - u_x \frac{\partial C}{\partial x'} - u_y \frac{\partial C}{\partial y'} = \phi \frac{\partial C}{\partial t} + q(x',y')\,(C_{in}-C) \qquad (5.23)$$

Introducing the dimensionless quantities as given by equations (5.12) and (5.13), Equation (5.23) can be written as

$$\frac{\partial C}{\partial t} = \frac{K_D}{L^2\phi}\frac{\partial^2 C}{\partial x^2} + \frac{K_D}{W^2\phi}\frac{\partial^2 C}{\partial y^2} - \frac{u_x}{L\phi}\frac{\partial C}{\partial x} - \frac{u_y}{W\phi}\frac{\partial C}{\partial y} - \frac{q(x,y)}{\phi}\,(C_{in}-C) \qquad (5.24)$$

Applying the method of orthogonal collocation on finite elements to Equation (5.24) and using the new independent variables ξ and v as given by Equation (5.16), one obtains

$$\frac{dC^{k,\ell}_{i,j}}{dt} = \frac{K_D}{L^2\phi\Delta x_k^2}\sum_{n=1}^{NPX} B_{i,n}\,C^{k,\ell}_{n,j} + \frac{K_D}{W^2\phi\Delta y_\ell^2}\sum_{n=1}^{NPY} B_{j,n}\,C^{k,\ell}_{i,n}$$

$$- \frac{u_x^{k,\ell}(x_i,y_j)}{L\phi\Delta x_k}\sum_{n=1}^{NPX} A_{i,n}\,C^{k,\ell}_{n,j} - \frac{u_y^{k,\ell}(x_i,y_j)}{W\phi\Delta y_\ell}\sum_{n=1}^{NPY} A_{j,n}\,C^{k,\ell}_{i,n}$$

$$- \frac{q(x_i,y_j)}{\phi}\,\{\,C_{in} - C^{k,\ell}_{i,j}\,\} \qquad (5.25)$$

Let

$$\frac{K_D}{L^2\phi\Delta x_k^2} = P_1 \qquad \frac{u_x^{k,\ell}(x_i,y_j)}{L\phi\Delta x_k} = P_3$$

$$\frac{K_D}{W^2\phi\Delta y_\ell^2} = P_2 \qquad \frac{u_y^{k,\ell}(x_i,y_j)}{W\phi\Delta y_\ell} = P_4$$

and

$$\frac{q(x_i,y_j)}{\phi} = P_5 \qquad (5.26)$$

Substituting Equation (5.26) into Equation (5.25), one obtains

$$\frac{dC_{i,j}^{k,\ell}}{dt} = P_1 \sum_{n=1}^{NPX} B_{i,n}\, C_{n,j}^{k,\ell} + P_2 \sum_{n=1}^{NPY} B_{j,n}\, C_{i,n}^{k,\ell}$$

$$- P_3 \sum_{n=1}^{NPX} A_{i,n}\, C_{n,j}^{k,\ell} - P_4 \sum_{n=1}^{NPY} A_{j,n}\, C_{i,n}^{k,\ell}$$

$$- P_5 \,(C_{in} - C_{i,j}^{k,\ell}) \qquad (5.27)$$

where i,j, k and ℓ are given by Equation (5.18).

The boundary conditions and the element interboundary conditions for Equation (5.27) are similar to Equations (5.20) to (5.22) and can be obtained by replacing P by C. In addition the concentration was assumed to be zero on the entire domain at t=0. Equation (5.27) together with the appropriate boundary conditions, describes the concentration field.

Two different methods, namely a 4th order Runge-Kutta method and a fully implicit method were used to solve Equation (5.27). A brief description of both the methods is given below.

5.3.2.1. Runge-Kutta Method

The k-values of the fourth order Runge-Kutta method are defined by

$$k_1 = h\, f\, (C_{i,j}^{k,\ell})$$

$$k_2 = h\, f\, (C_{i,j}^{k,\ell} + \frac{k_1}{2})$$

$$k_3 = h\, f\, (C_{i,j}^{k,\ell} + \frac{k_2}{2})$$

$$k_4 = h\, f\, (C_{i,j}^{k,\ell} + k_3) \qquad (5.28)$$

where f (C) is the right hand side of the Equation (5.27) evaluated at point (x_i,y_j) in the element kℓ.

The function value at a new time level (t+Δt) is given by

$$C_{i,j}^{t+\Delta t,k,\ell} = C_{i,j}^{t,k,\ell} + \frac{1}{6}(k_1+2k_2+2k_3+k_4) \qquad (5.29)$$

As Equation (5.27) is satisfied only at the interior collocation points, it is not possible to evaluate k_1, at the physical boundary and the element interboundaries. Consequently k_2 cannot be evaluated. To avoid this situation, the concentration C was assumed to remain constant at the boundary points during the time interval Δt. In other words, k_1, k_2, k_3 and k_4 were initially assumed to be zero at these boundary points.

Using Equations (5.28) and (5.29) the concentration at time (t+Δt) was evaluated for all interior collocation points. The appropriate conditions,as given by Equations (5.20) to (5.22), were then applied at the physical boundary and the element interboundaries at the(t+Δt) level. This resulted in three equations with three unknowns. Simultaneous solution of these equations gave the concentration at the new time level.

5.3.2.2 Total Implicit Method

The total implicit form of Equation (5.27) is given by

$$\frac{C_{i,j}^{t+\Delta t,k,\ell} - C_{i,j}^{t,k,\ell}}{\Delta t} = P_1 \sum_{n=1}^{NPX} B_{i,n} C_{n,j}^{t+\Delta t,k,\ell} + P_2 \sum_{n=1}^{NPY} B_{j,n} C_{i,n}^{t+\Delta t,k,\ell}$$

$$-P_3 \sum_{n=1}^{NPX} A_{i,n} C_{n,j}^{t+\Delta t,k,\ell} - P_4 \sum_{n=1}^{NPY} A_{j,n} C_{i,n}^{t+\Delta t,k,\ell} - P_5(C_{in} - C_{i,j}^{t+\Delta t,k,\ell}) \qquad (5.30)$$

where i,j,k and ℓ are given by Equation (5.18). Equation (5.30) provided the appropriate equations for all the interior collocation points. Equations (5.20) to (5.22) with P replaced by C provided the equations for the boundary points (both the physical and the element interboundary points). Thus the set of Equations (5.20) to (5.22) and (5.30) define the system completely. The simultaneous solution of the above mentioned set of equations was obtained by a direct method and the solution gave the concentration field at the new time level, $t+\Delta t$.

5.4 Determination of Source Term

The source term, q, used in Equations (5.19) and (5.27) is the amount injected or produced in cubic centimeters per unit volume of the formation surrounding the well under consideration per second. It was converted to a more useful quantity Q which is the total amount injected or produced in cubic centimeters per second. Q was evaluated by integrating $\int q \, dV$ over the element under consideration using a quadrature approach. The following formula was used to obtain Q from q:

$$Q = q \, w_2^2 \, (\Delta y_\ell \, \Delta x_k) \, (WL) \, (S) \tag{5.31}$$

where S is the thickness of the formation. The weighting factors w are listed in Appendix B. W_2 is the weighting factor for the second collocation point where the injection well is located.

5.5 Computational Scheme

For a given initial concentration profile, Equation (5.3) was used to evaluate the viscosity distribution in the formation. The pressure was then evaluated using Equation (5.19). Equations (5.6) and (5.7) were then used to evaluate the oil-solvent mixture velocity at the collocation points, using the viscosity and the velocity field at the old time level, the concentration at the new time level was evaluated by solving Equation (5.27). Once the concentration profile was known, the whole cycle was repeated for a subsequent time interval.

The flow chart for the computation scheme is shown in Figure 14. Sections one and two refer to the solution scheme for Equations (5.19) and (5.27), respectively.

5.6 Results and Discussion

As stated earlier, two different locations were used for the production well. In one scheme the production and the injection wells are symmetric about the line $x = 0.5$. This situation will be referred to as 'Configuration One'. For the other scheme the production and the injection wells are diagonally opposite. This situation will be referred to as 'Configuration Two'. Figure 9 shows both the configurations. Figures 10 to 13 show the locations of the production and the injection wells for N=2,3,4 and 5, respectively.

Two different initial viscosities of the oil were used (100 and 10,000 cp). Initially, the oil viscosity was assumed to be constant for the entire formation and pure solvent was used for injection.

A list of all the physical data used in this problem is provided in Appendix H.

5.6.1 Velocity Results

Due to the nature of the boundary conditions, the continuity equation has an infinite number of solutions. To obtain a unique solution, the pressure at one collocation point was specified. The

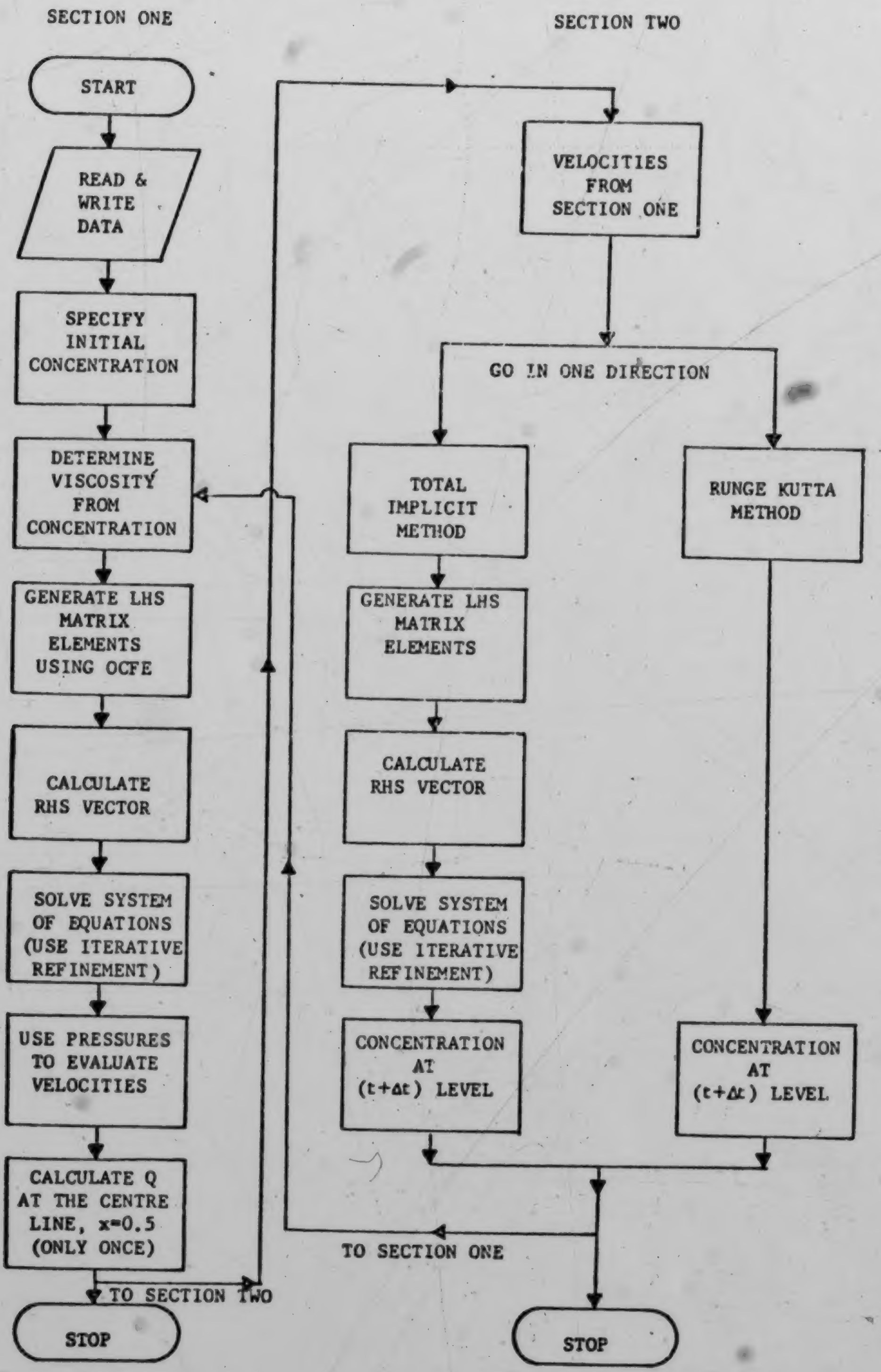

SECTION ONE
SECTION TWO
START
READ & WRITE DATA
SPECIFY INITIAL CONCENTRATION
DETERMINE VISCOSITY FROM CONCENTRATION
GENERATE LHS MATRIX ELEMENTS USING OCFE
CALCULATE RHS VECTOR
SOLVE SYSTEM OF EQUATIONS (USE ITERATIVE REFINEMENT)
USE PRESSURES TO EVALUATE VELOCITIES
CALCULATE Q AT THE CENTRE LINE, x=0.5 (ONLY ONCE)
TO SECTION TWO
STOP
VELOCITIES FROM SECTION ONE
GO IN ONE DIRECTION
TOTAL IMPLICIT METHOD
RUNGE KUTTA METHOD
GENERATE LHS MATRIX ELEMENTS
CALCULATE RHS VECTOR
SOLVE SYSTEM OF EQUATIONS (USE ITERATIVE REFINEMENT)
CONCENTRATION AT (t+Δt) LEVEL
CONCENTRATION AT (t+Δt) LEVEL
TO SECTION ONE
STOP

FIGURE 14. Flow Chart for the Computational Scheme for Porous Media Problem

pressure at all other collocation points was evaluated with respect to this reference pressure. The collocation points where the pressure was specified are shown in Figures 10 to 13 for N=2,3,4 and 5, respectively. The reference pressure collocation point was assumed to have a pressure of 1 psi.

5.6.1.1 Configuration One

In this configuration the production and the injection wells are symmetric about the line x = 0.5. Tables E.1 to E.8 refer to this configuration. These tables show the pressure and the velocities along the x and y-directions at the collocation points for N=2,3,4 and 5. It can be shown that when q, k_p and the dimensions of the formation are kept constant, a change in the initial viscosity of the oil in the formation changes the pressure gradient between any two points proportionally. Therefore, the velocity at any collocation point is independent of the viscosity of the oil although the pressure at a collocation point is a function of viscosity. Such an observation can be made in Tables E.1 to E.8 where two different viscosities were used.* Therefore in the next sections, no reference to the viscosity will be made while dealing with the velocities.

Due to symmetry there is no flow in the y-direction along the line x = 0.5 and the pressure at the collocation points along this line should be the same. Tables E.1 to E.8 indicate that the pressure at the collocation points along the line x = 0.5 are constant and therefore the flow is in the x-direction only. To check the validity of the numerical results presented in Tables E.1 to E.8, the velocities in the x-direction along the line x = 0.5 were integrated to obtain the total amount of fluid crossing the line. A quadrature approach was utilized for the integration. The following formula was used to obtain the total flow rate along the line x = 0.5.

---

*Tables E.1 to E.2 provide velocities for N=2 and for two different sets of viscosities of the oil-solvent mixture, namely, (100 and 10,000 cp). The velocity at any particular collocation point in both the tables is identical. Similar conclusions apply to Tables E.3 and E.4 (for N=3), Tables E.5 and E.6 (for N=4) and Table E.7 and E.8 (for N=5).

$$Q_C = WS \sum_{\ell=1}^{NEY} \Delta y_\ell \sum_{j=2}^{NPY-1} U^{1,\ell}_{x_{NPX,j}} \; w_j \qquad (5.32)$$

where w is the weighting factor obtained from Appendix B and S is the thickness of the formation.

Since the fluid and the formation are incompressible and there is no flow in y-direction, the value of Q_C obtained from Equation (5.32) should be equal to the total solvent injection rate Q defined by Equation (5.31). Table 4 shows the value of Q_C for the various numbers of interior collocation points. The value of Q_C is in excellent agreement with the value of Q. The velocities obtained for this injection rate are of the order of 10^{-5} to 10^{-7} cm/sec. These velocities compare favourably with those presented by Settari et al (1976) for the same injection rate (10^{-2} to 10^{-4} ft/day).

Table 4

Comparison of Evaluated and Injected Q.

Injection rate Q = .3277 cm^3/sec

Evaluated Q_C Using Quadrature Approach

	Configuration 1				
Q_C cm^3/sec	N = 2	N = 3	N = 4	N = 5	
	.327690	.327698	.327702	.3277	

It was noted in Chapter 4 that,for odd number of collocation points, the middle collocation point is always at 0.5. Therefore for elements of equal size and for two different sets of an odd number of collocation points, there exists at least 4 collocation points (at the centre of each of the 4 elements) which are identically located. Table 5 gives the corresponding collocation points located at the centre of each of the 4 elements for N=3 and 5.

Table 5

Collocation Point Number of the Point Located at the Centre of Each Element for N=3 and 5.

	N = 3	N = 5
Collocation point Number	18	40
	22	46
	51	115
	55	121

Comparison of the results at these points leads to some interesting conclusions. Table 6 presents the comparison of the pressures for N=3 and 5 at the 4 collocation points located at the centre of each element. The results are in excellent agreement. The slight difference in the values could be attributed to the fact that the injection, production and reference wells are not exactly at the same positions for the two cases.

Table 6

Comparison of the Pressures at the Centre of Each Element for N=3 and 5.

	Collocation Point Number	Viscosity 100 cp Pressure psi	Viscosity 10,000 cp Pressure psi
N = 3	18	1.20366	21.36594
N = 5	40	1.19999	20.99703
N = 3	22	.999463	.46286
N = 5	46	.99466	.46466
N = 3	51	.63496	-35.50453
N = 5	115	.63863	-35.13795
N = 3	55	.84399	-14.60141
N = 5	121	.84396	-14.6055

Figure 15 shows the directions (not magnitudes) of the resultant velocities at the collocation points for N=3. These velocities were taken from Table E.3. As expected, the general direction of the velocity profile is towards the production well.

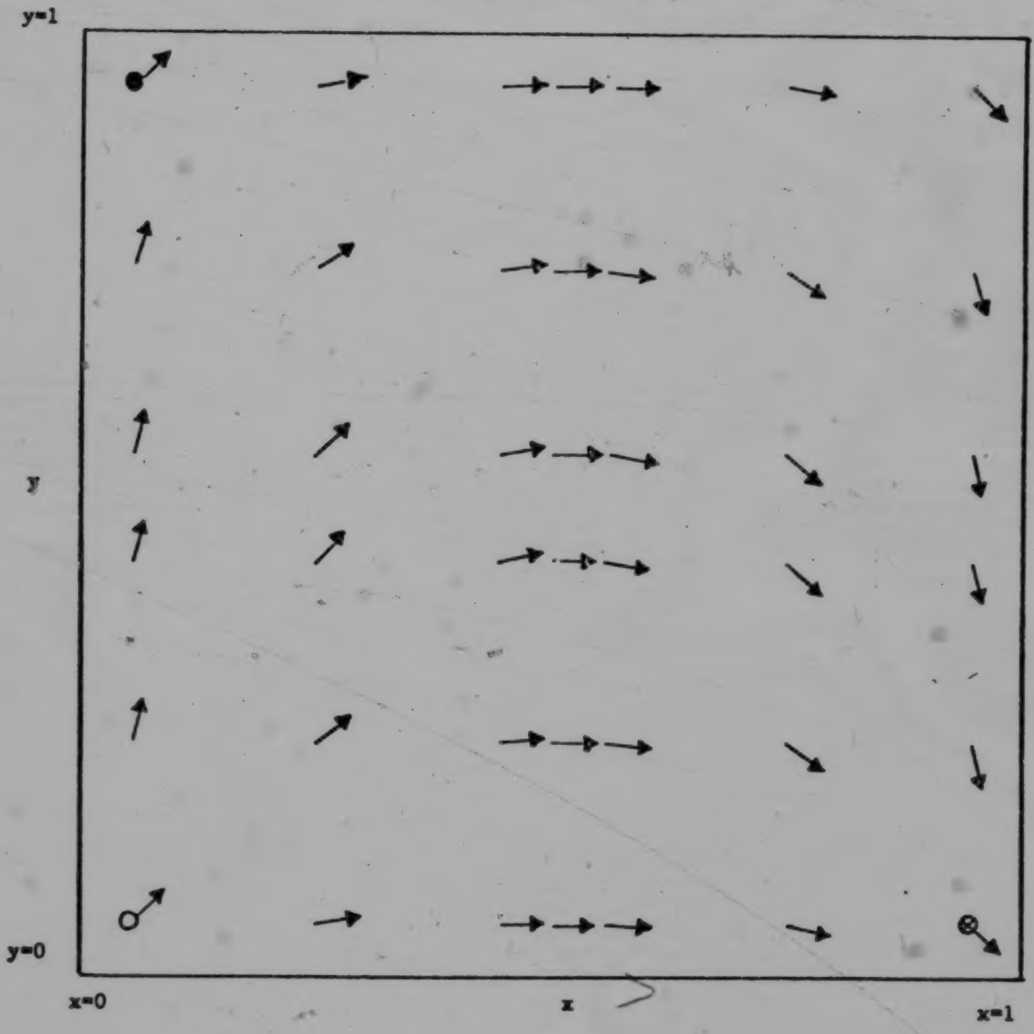


FIGURE 15. Configuration One: Direction of the Resultant Velocities for N=3

5.6.1.2 Configuration Two

In this configuration the production and the injection wells are diagonally opposite. The diagonal joining of the two wells becomes the symmetry line (Figure 16).

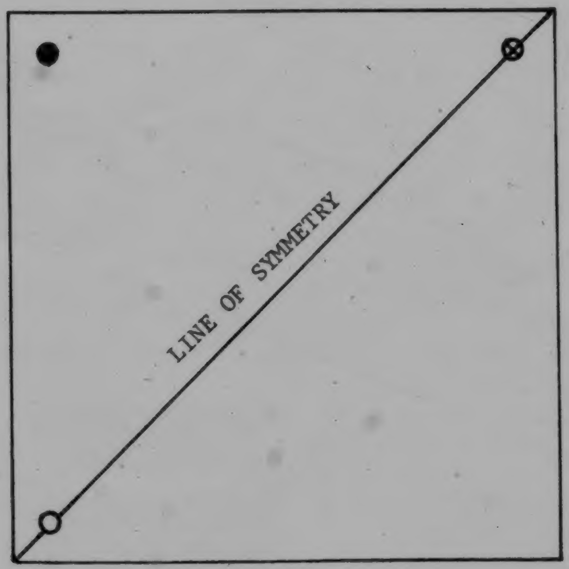


FIGURE 16.

Tabulation of the pressures and the velocities at the collocation points for various N are given in Tables E.9 to E.16. Any two collocation points which are symmetrically located with respect to the line of symmetry should have the same pressure. An examination of Tables E.9 to E.16 indicates that the pressure at corresponding points is the same. Moreover, any two symmetrically located collocation points also exchange velocity components, i.e. the x-velocity component at one collocation point becomes the y-velocity component at the corresponding symmetric collocation point and vice versa. Furthermore any collocation point on any of the two diagonals has identical velocity components. This means that the direction of the velocity is parallel to the line of symmetry. This flow pattern is expected for this type of configuration when the mobility ratio (K_p/μ) is constant throughout the formation. Figure 17 shows the directions (not magnitudes) of resultant velocities at the collocation points for N=3. These velocities were taken from Table E.11.

5.6.2 Concentration Results

The orthogonal collocation on finite element formulation of the convection-diffusion equation is given by Equation (5.27). Two different dispersion coefficients were used (K_D=0.0001075 and 0.01075 cm^2/s). Pure solvent was injected through the injection well. Initial concentration of the solvent was assumed to be zero for the entire formation.

The set of first order differential equations obtained from Equation (5.27) together with the boundary conditions were solved using two different methods of solution, namely the Runge-Kutta method and the Total Implicit method. Unrealistic concentration profiles were obtained from both the methods of solution. The tables of Appendix F† present the concentration profiles obtained from both the methods of solution.

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† Tables F.1 to F.4 refer to 'Configuration One'. Tables F.1 and F.2 present concentration profiles for N=3 and K_D = 0.0001075 and 0.01075 cm^2/s respectively. Tables F.3 and F.4 present concentration profiles for N=5 and K_D = .0001075 and .01075 cm^2/s, respectively. Tables F.5 to F.8 refer to 'Configuration Two'. Tables F.5 and F.6 show the concentration profiles for N=3 and K_D = .0001075 and 0.01075 cm^2/s, respectively. Tables F.7 and F.8 provide the concentration profiles for N=5 and K_D = 0.0001075 and 0.01075 cm^2/s, respectively.

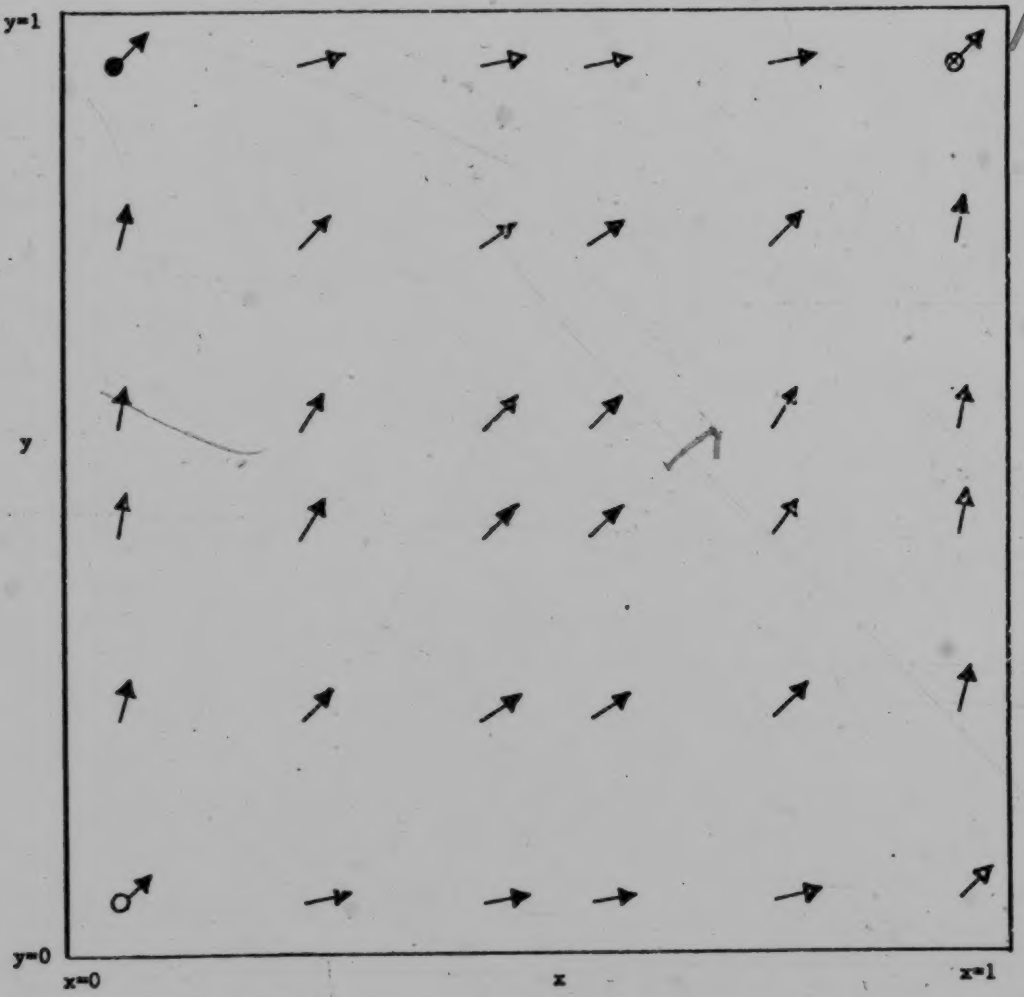


FIGURE 17. Configuration Two: Direction of the Resultant Velocities for N=3

After the completion of the first iteration, negative values of concentration at several collocation points were obtained. Such negative values of solvent concentration are physically unrealistic. However, these negative concentrations do satisfy the convection - diffusion equation as is shown for two different cases (N=3 and 5) in Appendix G. Also an increase in the number of interior collocation points from 3 to 5 does not improve the concentration profile.

The fourth order Runge-Kutta method of integration was first used to solve Equation (5.27). It was suspected that the negative values of concentration were due to the method of solution since the Runge-Kutta method is a single step explicit method. A Total Implicit method was then used to solve all the equations simultaneously. However, no realistic profile could be obtained and indeed the two methods gave approximately the same values of concentration at the collocation points. (See Tables F.1 to F.8).

To increase the effect of the diffusion terms, a large value of K_D was also used (K_D = 0.01075 cm^2/s). Tables F.2 to F.4 for 'Configuration One' and Tables F.6 to F.8 for 'Configuration Two' present results for K_D = 0.01075 cm^2/sec. However, such an increase in K_D did not give a realistic concentration profile.

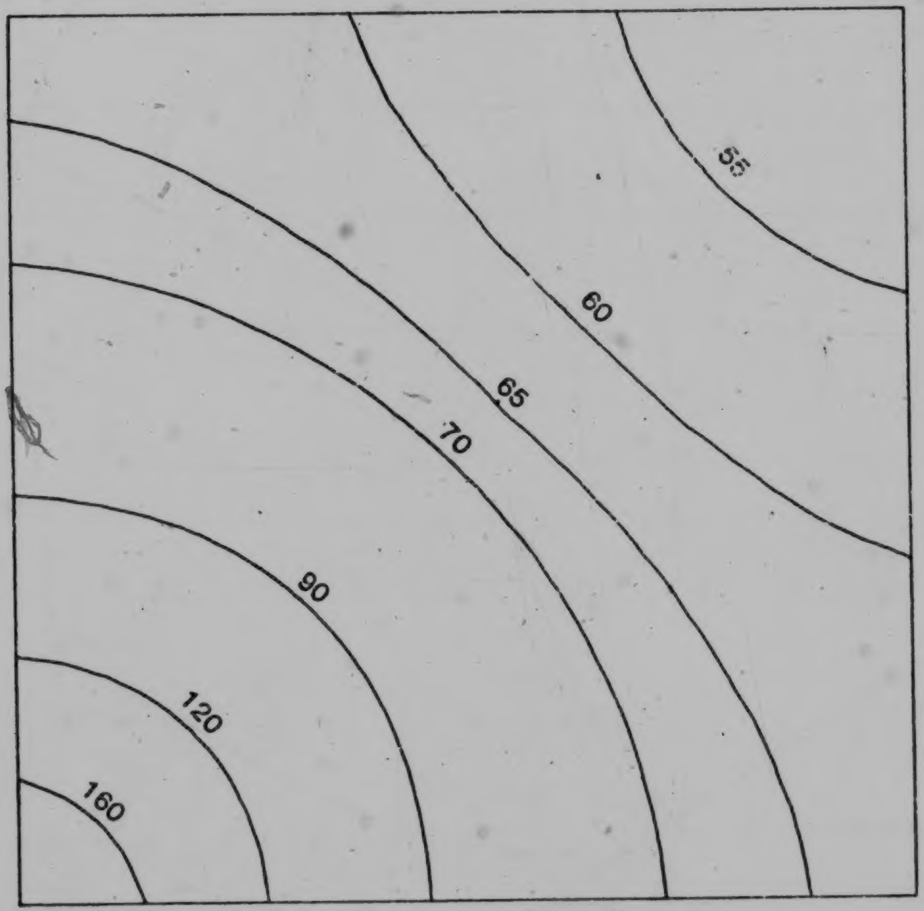
It was thought that the negative values of concentration could be due to initial instability of the numerical method. It was hoped that after a few time steps a realistic concentration profile would be obtained. A number of time steps were used but no realistic profile could be obtained. Some other schemes were tried to obtain a realistic profile. The important schemes attempted are listed below.

1. At the end of one complete time step all the negative values of concentration were equated to zero. Such a method ensured that no negative value of concentration was used in the determination of the viscosity from Equation (5.3). A few time steps were made using this approach without any success

2. It was found that the negative values of concentration at the boundaries were mainly caused by the continuity of the first derivative condition at the element interboundaries. To check the negative values of concentration at the two physical boundaries (i.e. at $x = 1$ and $y = 1$) different boundary conditions were used. The concentration at these two boundaries was assumed to be zero. Such alternative boundary conditions are justified since there is practically no change in the concentration for a large period of time at these two physical boundaries. This approach, although removed the negative values of the concentration from the boundaries did not give a realistic concentration profile.

The various schemes described above gave some insight into the method of orthogonal collocation on finite elements. Since only two elements were used in each direction, there was only one element interboundary per direction. When the continuity of the first derivative was imposed on such an element interboundary, the resulting equations contained the values of concentration of the collocation points situated on the two opposite physical boundaries. Since there is a considerable difference in the concentration change between any two end points in one direction, only two elements per direction are not sufficient to accommodate the large concentration gradient in the formation. A better approximation of the physical situation could be achieved by using more number of elements. However, extremely high computational demands prevented the use of a larger number of elements.

On further increasing the diffusion coefficient to 1.075 cm^2/s, the negative concentration values disappeared and a solution was obtained for a total of 50 time steps for N=3. A contour for the concentration profile after the 50 time steps is shown in Figure 18. The concentration profile was found to be insensitive to the location of the production well. The concentration profile was always symmetric about the line x=y. Therefore, this type of a profile indicates that the effect of the convection terms in the convection diffusion equation is negligible and for a large value of the diffusion coefficient, the convection-diffusion equation degenerates to a diffusion equation.



Concentration = contour value * 10^{-4}

FIGURE 18. Contour for Concentration Profile After 50 Time Steps for K_D = 1.075 cm^2/s

CHAPTER 6

CONCLUSION AND RECOMMENDATION

6.1 Conclusion:

6.1.1 Finned Tube Problem

The results indicate that the OCFE method on the basis of equivalent number of interior collocation points is superior to the finite-difference method. However, the finite difference approach is simple to apply and the advantage gained by using the OCFE method is lost in the relatively greater programming complexity of the method and its higher computational cost.

6.1.2 Porous Media Problem

Solution of the continuity equation provided excellent mass conservation. However, realistic concentration profiles from the convection-diffusion equation could be generated only for a very high value of the diffusion coefficient.

6.2 Recommendation:

In any future work on the convection-diffusion equation, more elements in each direction should be used.

It is recommended that in any future simulation of porous media problems using the OCFE method, a better algorithm should be developed to minimize the extraneous zeros introduced in the matrix. Such an algorithm would permit the use of a larger number of elements and collocation points at a lesser computational cost.

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APPENDIX A

Computer Program to Generate Matrices A, B and w.

The accompanying computer program generates matrices for the orthogonal collocation method for a general polynomial approximation satisfying Equation (3.13a).The first and the second derivative approximations are represented by A and B matrices, respectively. The program can be used for up to 18 interior collocation points. With minor modifications it can be used for any number of collocation points. Appendix B contains the tabulation of the matrices up to 10 interior collocation points. Double precision arithmetic was used to insure a high accuracy.

```
C=====================================================================C
C                                                                     C
C          MATRICES FOR THE METHOD OF ORTHOGONAL COLLOCATION          C
C                                                                     C
C                 (GENERAL POLYNOMIAL APPROXIMATION)                  C
C                                                                     C
C=====================================================================C
C
C
C     THE PROGRAM USES SUBROUTINE MINV WHICH IS AVAILABLE
C     IN THE SSP LIBRARY.
C
C     INPUT DATA REQUIRED
C
C         1. TOTAL NUMBER OF INTERIOR COLLOCATION POINTS.
C         2. COLLOCATION ABSCISSAS.
C
C
C     N: THE NUMBER OF INTERIOR COLLOCATION POINTS
C     X : COLLOCATION ABSCISSAS
C
C
      IMPLICIT REAL * 8(A - H, O - Z), INTEGER(I - N)
      DIMENSION Q(20, 20), P(20, 20), X(20), W(1, 20), L(20), M(20), QI(
     &20, 20), C(20, 20), F(1, 20), V(50), A(20, 20), B(20, 20), D(20,
     &20)
C
C      READ THE NUMBER OF INTERIOR COLLOCATION POINTS
C
      READ (5,140) N
      WRITE (6,170)
      N2 = N + 2
      WRITE (6,260) N
C
C      READ THE COLLOCATION ABSCISSAS
C
      READ (5,130) (X(I), I = 1, N2)
      DO 20 I = 1, N2
      DO 20 J = 1, N2
      IF (X(J) .LT. 1.E - 40) GO TO 10
      Q(J, I) = X(J) ** (I - 1)
      C(J, I) = (I - 1) * ((X(J)) ** (I - 2))
      D(J, I) = (I - 1) * (I - 2) * ((X(J)) ** (I - 3))
      GO TO 20
   10 Q(J, I) = 0.D0
      Q(1, 1) = 1.D0
      C(J, I) = 0.D0
      C(J, 2) = 1.D0
      D(J, I) = 0.D0
      D(J, 3) = 2.D0
   20 CONTINUE
      WRITE (6,210)
      DO 30 IJ = 1, N2
      WRITE (6,120) (Q(IJ, K), K = 1, N2)
   30 CONTINUE
      DO 40 I = 1, N2
      DO 40 J = 1, N2
      K = I + N2 * (J - 1)
      V(K) = Q(I, J)
   40 CONTINUE
```

```
C
C     SUBROUTINE MINV CALCULATES MATRIX INVERSE
C
      CALL MINV(V, N2, DET, L, M)
      DO 50 I = 1, N2
      DO 50 J = 1, N2
      K = I + N2 * (J - 1)
      QI(I, J) = V(K)
   50 CONTINUE
      WRITE (6,220)
      DO 60 IJ = 1, N2
      WRITE (6,120) (QI(IJ, K), K = 1, N2)
   60 CONTINUE
      ZI = 0.D0
      DO 70 IJ = 1, N2
      ZI = ZI + 1.D0
      F(1, IJ) = 1.D0 / ZI
   70 CONTINUE
      WRITE (6,230)
      WRITE (6,150) (F(1, II), II = 1, N2)
      WRITE (6,240)
      DO 80 IJ = 1, N2
      WRITE (6,120) (C(IJ, K), K = 1, N2)
   80 CONTINUE
      WRITE (6,250)
      DO 90 IJ = 1, N2
      WRITE (6,120) (D(IJ, K), K = 1, N2)
   90 CONTINUE
      CALL MMPL(C, QI, A, N2, N2, N2)
      CALL MMPL(D, QI, B, N2, N2, N2)
      CALL MPLM(F, QI, W, 1, N2, N2)
      WRITE (6,180)
      WRITE (6,150) (W(1, J), J = 1, N2)
      WRITE (6,190)
      DO 100 IJ = 1, N2
      WRITE (6,160) (A(IJ, K), K = 1, N2)
  100 CONTINUE
      WRITE (6,200)
      DO 110 IJ = 1, N2
      WRITE (6,160) (B(IJ, K), K = 1, N2)
  110 CONTINUE
      STOP
C
  120 FORMAT (8F15.6)
  130 FORMAT (D12.10)
  140 FORMAT (I2)
  150 FORMAT (F10.6)
  160 FORMAT (8F15.6)
  170 FORMAT ( / /, 'ORTHOGONAL COLLOCATION METHOD', / / )
  180 FORMAT ( / /, 'VECTOR W', / )
  190 FORMAT ( / /, 'MATRIX A', / )
  200 FORMAT ( / /, 'MATRIX B', / )
  210 FORMAT (' Q MATRIX', / )
  220 FORMAT ( / /, 'INVERSE OF Q MATRIX', / )
  230 FORMAT ( / /, 'VECTOR F', / )
  240 FORMAT ( / /, 'MATRIX C', / )
  250 FORMAT ( / /, 'MATRIX D', / )
  260 FORMAT ( / /, 'NUMBER OF INTERIOR COLLOOCATION POINTS = ', I2, / )
C
      END
```

```
C
C
      SUBROUTINE MMPL(A, B, C, L, M, N)
      IMPLICIT REAL * 8(A - H, O - Z), INTEGER(I - N)
      DIMENSION A(20, 20), B(20, 20), C(20, 20)
      DO 20 J = 1, N
      DO 20 I = 1, L
      C(I, J) = 0.D0
      DO 10 K = 1, M
      C(I, J) = C(I, J) + A(I, K) * B(K, J)
   10 CONTINUE
   20 CONTINUE
      RETURN
      END
C
C
      SUBROUTINE MPLM(A, B, C, L, M, N)
      IMPLICIT REAL * 8(A - H, O - Z), INTEGER(I - N)
      DIMENSION A(1, 20), B(20, 20), C(1, 20)
      DO 20 J = 1, N
      C(1, J) = 0.D0
      DO 10 K = 1, M
      C(1, J) = C(1, J) + A(1, K) * B(K, J)
   10 CONTINUE
   20 CONTINUE
      RETURN
      END
```

APPENDIX B

Tabulation of Matrices A, B and w.

```
     NUMBER OF INTERIOR COLLOCATION POINTS= 1

VECTOR w (WEIGHTING FACTOR)

 0.166667
 0.666667
 0.166667

MATRIX A

    -3.000000       4.000000      -1.000000
    -1.000000       0.0            1.000000
     1.000000      -4.000000       3.000000

MATRIX B

     4.000000      -8.000000       4.000000
     4.000000      -8.000000       4.000000
     4.000000      -8.000000       4.000000
```

```
NUMBER OF INTERIOR COLLOCATION POINTS =  2

VECTOR w (WEIGHTING FACTOR)

-0.000000
 0.500000
 0.500000
-0.000000

MATRIX A

   -7.000000      8.196152     -2.196152      1.000000
   -2.732051      1.732051      1.732051     -0.732051
    0.732051     -1.732051     -1.732051      2.732051
   -1.000000      2.196152     -8.196152      7.000000

MATRIX B

   24.000000    -37.176915     25.176915    -12.000000
   16.392305    -24.000000     12.000000     -4.392305
   -4.392305     12.000000    -24.000000     16.392305
  -12.000000     25.176915    -37.176915     24.000000
```

```
NUMBER OF INTERIOR COLLOCATION POINTS =  3

VECTOR W (WEIGHTING FACTOR)

 0.000000
 0.277778
 0.444444
 0.277778
 0.000000

MATRIX A

    -13.000000       14.788306       -2.666667        1.878361       -1.000000
     -5.323790        3.872983        2.065591       -1.290994        0.676210
      1.500000       -3.227486       -0.000000        3.227486       -1.500000
     -0.676210        1.290994       -2.065591       -3.872983        5.323790
      1.000000       -1.878361        2.666667      -14.788306       13.000000

MATRIX B

     84.000000     -122.063167       58.666667      -44.603500       24.000000
     53.237900      -73.333333       26.666667      -13.333333        6.762100
     -6.000000       16.666667      -21.333333       16.666667       -6.000000
      6.762100      -13.333333       26.666667      -73.333333       53.237900
     24.000000      -44.603500       58.666667     -122.063167       84.000000
```

```
  NUMBER OF INTERIOR COLLOCATION POINTS =  4

VECTOR w (WEIGHTING FACTOR)

-0.000000
 0.173927
 0.326073
 0.326073
 0.173927
-0.000000


MATRIX A

   -21.000000     23.630444     -3.679876      1.812554     -1.763122      1.000000
    -8.778306      6.664000      2.840481     -1.232462      1.161256     -0.654970
     2.495250     -5.184822      0.768829      2.941340     -2.249653      1.229056
    -1.229056      2.249653     -2.941340     -0.768829      5.184822     -2.495250
     0.654970     -1.161256      1.232462     -2.840481     -6.664000      8.778306
    -1.000000      1.763122     -1.812554      3.679876    -23.630444     21.000000


MATRIX B

   220.000000   -311.798359    132.253155    -70.716581     70.261785    -40.000000
   135.863821   -183.029674     59.659350    -20.530641     18.174258    -10.137114
   -11.285451     31.822354    -36.970326     21.825742    -10.951063      5.558744
     5.558744    -10.951063     21.825742    -36.970326     31.822354    -11.285451
   -10.137114     18.174258    -20.530641     59.659350   -183.029674    135.863821
   -40.000000     70.261784    -70.716581    132.253155   -311.798359    220.000000
```

```
NUMBER OF INTERIOR COLLOCATION POINTS =  5

VECTOR W (WEIGHTING FACTOR)

 0.000000
 0.118463
 0.239314
 0.284444
 0.239314
 0.118463
 0.000000

MATRIX A

   -31.000000     34.699724     -5.031518      2.133333     -1.509422      1.707884     -1.000000
   -13.096091     10.134081      3.879729     -1.446279      0.987518     -1.103534      0.644576
     3.732156     -7.625116      1.516706      3.412150     -1.857116      1.940842     -1.119622
    -1.875000      3.368054     -4.043058      0.000000      4.043058     -3.368054      1.875000
     1.119622     -1.940842      1.857116     -3.412150     -1.516706      7.625116     -3.732156
    -0.644576      1.103534     -0.987518      1.446279     -3.879729    -10.134081     13.096091
     1.000000     -1.707884      1.509422     -2.133333      5.031518    -34.699724     31.000000

MATRIX B

   479.999999   -671.968385    268.346906   -123.733333     89.659712   -102.304900     60.000000
   292.915037   -390.420938    120.839145    -35.697494     22.749456    -24.802174     14.416968
   -21.024726     59.816814    -66.912395     35.697494    -12.531160     11.261251     -6.307280
     7.500000    -14.867044     30.033711    -45.333333     30.033711    -14.867044      7.500000
    -6.307280     11.261251    -12.531160     35.697494    -66.912395     59.816814    -21.024726
    14.416968    -24.802174     22.749456    -35.697494    120.839145   -390.420939    292.915038
    60.000000   -102.304899     89.659712   -123.733333    268.346906   -671.968385    480.000000
```

```
NUMBER OF INTERIOR COLLOCATION POINTS =  6

VECTOR W (WEIGHTING FACTOR)

-0.000000
 0.085662
 0.180381
 0.233957
 0.233957
 0.180381
 0.085662
 0.000000

MATRIX A

   -43.000000     47.989761     -6.684145      2.616714     -1.608497      1.363179     -1.677011      1.000000
   -18.277284     14.290654      5.151964     -1.772041      1.049905     -0.876916      1.072421     -0.638703
     5.213764    -10.551525      2.349705      4.163815     -1.955470      1.512380     -1.795975      1.063306
    -2.636938      4.688723     -5.379349      0.506053      4.190778     -2.526326      2.777989     -1.620929
     1.620929     -2.777989      2.526326     -4.190778     -0.506053      5.379349     -4.688723      2.636938
    -1.063306      1.795975     -1.512380      1.955470     -4.163815     -2.349705     10.551525     -5.213764
     0.638703     -1.072421      0.876916     -1.049905      1.772041     -5.151964    -14.290654     18.277284
    -1.000000      1.677011     -1.363179      1.608497     -2.616714      6.684145    -47.989761     43.000000

MATRIX B

   924.000000  -1284.565562    495.918786   -211.290168    133.136276   -113.951015    140.751683    -84.000000
   560.220666   -742.282115    223.220705    -60.862959     33.593730    -27.264391     32.951373    -19.577009
   -37.055750    106.006704   -116.339981     58.979791    -17.882184     11.681881    -12.947770      7.557229
    11.184594    -22.284687     45.473410    -65.377904     39.366746    -13.787161     12.300187     -6.875184
    -6.875184     12.300187    -13.787161     39.366746    -65.377904     45.473410    -22.284687     11.184594
     7.557229    -12.947770     11.681881    -17.882184     58.979791   -116.339981    106.006784    -37.055750
   -19.577009     32.951373    -27.264391     33.593730    -60.862959    223.220705   -742.282115    560.220666
   -84.000000    140.751683   -113.951015    133.136276   -211.290168    495.918787  -1284.565563    924.000000
```

```
NUMBER OF INTERIOR COLLOCATION POINTS =  7

VECTOR W (WEIGHTING FACTOR)

-0.000000
 0.064742
 0.139853
 0.190915
 0.208980
 0.190915
 0.139853
 0.064742
-0.000000

MATRIX A

   -57.000000     63.498187     -8.626161      3.218562     -1.828571      1.360266     -1.280249      1.657966
    -1.000000
   -24.321936     19.136365      6.647609     -2.178562      1.192380     -0.873485      0.816194     -1.053621
     0.635057
     6.941065    -13.564880      3.294731      5.110136     -2.212009      1.495064     -1.348561      1.714608
    -1.030156
    -3.520456      6.221143     -6.946419      0.971747      4.712161     -2.463994      2.032302     -2.494341
     1.487856
     2.187500     -3.724038      3.288625     -5.153701      0.000000      5.153701     -3.288625      3.724038
    -2.187500
    -1.487856      2.494341     -2.032302      2.463994     -4.712161     -0.971747      6.946419     -6.221143
     3.520456
     1.030156     -1.714608      1.348561     -1.495064      2.212009     -5.110136     -3.294731     13.964880
    -6.941065
    -0.635057      1.053621     -0.816194      0.873485     -1.192380      2.178562     -6.647609    -19.136365
    24.321936
     1.000000     -1.657966      1.280249     -1.360266      1.828571     -3.218562      8.626161    -63.498187
    57.000000

MATRIX B

  1624.000002  -2247.983161    849.885999   -345.247862    201.142857   -151.200061    143.007875   -185.605649
   112.000000
   980.780842  -1294.766277    382.521441    -99.420096     50.660910    -36.009313     33.169047    -42.545185
    25.608631
   -61.680334    177.081949   -192.199515     94.564909    -26.508061     15.063785    -12.523525     15.355060
    -9.154268
    16.858608    -33.715020     69.272481    -97.034209     55.600997    -16.931290     11.034810    -12.211361
     7.124984
    -8.750000     15.694897    -17.739645     50.794748    -80.000000     50.794748    -17.739645     15.694897
    -8.750000
     7.124984    -12.211361     11.034810    -16.931290     55.600997    -97.034209     69.272481    -33.715020
    16.858608
    -9.154268     15.355060    -12.523525     15.063785    -26.508061     94.564909   -192.199515    177.081949
   -61.680334
    25.608631    -42.545185     33.169047    -36.009313     50.660910    -99.420096    382.521441  -1294.766277
   980.780842
   112.000000   -185.605649    143.007875   -151.200061    201.142857   -345.247862    849.885999  -2247.983161
  1624.000002
```

```
NUMBER OF INTERIOR COLLOCATION POINTS = 8

VECTOR W (WEIGHTING FACTOR)

-0.000000
 0.050614
 0.111191
 0.156853
 0.181342
 0.181342
 0.156853
 0.111191
 0.050614
-0.000000

MATRIX A

   -73.000000     81.223956    -10.852921      3.924518     -2.122025      1.464189     -1.220595      1.228254
    -1.645377      1.000000
   -31.230068     24.672354      8.362857     -2.655769      1.383075     -0.939430      0.777088     -0.778824
     1.041352     -0.632636
     8.914467    -17.865476      4.361442      6.224194     -2.560891      1.602304     -1.276352      1.255230
    -1.663792      1.008874
    -4.527522      7.968484     -8.741958      1.452117      5.440369     -2.625140      1.902832     -1.792652
     2.331610     -1.408140
     2.827012     -4.792190      4.153551     -6.282481      0.379644      5.451533     -3.031485      2.598803
    -3.255013      1.950627
    -1.950627      3.255013     -2.598803      3.031485     -5.451533     -0.379644      6.282481     -4.153551
     4.792190     -2.827012
     1.408140     -2.331610      1.792652     -1.902832      2.625140     -5.440369     -1.452117      8.741958
    -7.968484      4.527522
    -1.008874      1.663792     -1.255230      1.276352     -1.602304      2.560891     -6.224194     -4.361442
    17.865476     -8.914467
     0.632636     -1.041352      0.778824     -0.777088      0.939430     -1.383075      2.655769     -8.362857
   -24.672354     31.230068
    -1.000000      1.645377     -1.228254      1.220595     -1.464189      2.122025     -3.924518     10.852921
   -81.223956     73.000000

MATRIX B

  2663.999993  -3677.014145   1371.026631   -539.893947    299.420769   -208.822661    175.006359   -176.590563
   236.867562   -144.000000
  1604.764018  -2113.392370    617.104369   -155.482609     75.368841    -49.641410     40.437206    -40.203940
    53.554051    -32.508156
   -97.606537    280.907822   -302.743370    146.117517    -39.042583     20.516053    -14.994770     14.100436
   -18.300958     11.046391
    25.020304    -50.171959    103.580096   -142.810312     79.411961    -22.435091     12.767810    -10.629524
    13.048494     -7.781758
   -11.701791     21.036169    -23.939119    -68.688100   -105.053922     63.577703    -19.405437     12.579501
   -13.855396      8.074191
     8.074191    -13.855396     12.579501    -19.405437     63.577703   -105.053922     68.688100    -23.939119
    21.036169    -11.701791
    -7.781758     13.048494    -10.629524     12.767810    -22.435091     79.411961   -142.810332    103.580096
   -50.171959     25.020304
    11.046391    -18.300958     14.100436    -14.994770     20.516053    -39.042583    146.117517   -302.743370
   280.907822    -97.606537
   -32.508156     53.554051    -40.203940     40.437206    -49.641410     75.368841   -155.482609    617.104369
 -2113.392370   1604.764018
  -144.000000    236.867562   -176.590563    175.006359   -208.822661    299.420769   -539.893947   1371.026631
 -3677.014145   2663.999998
```

```
NUMBER OF INTERIOR COLLOCATION POINTS = 9

VECTOR W (WEIGHTING FACTOR)

-0.000000
 0.040637
 0.090324
 0.130305
 0.156174
 0.165120
 0.156174
 0.130305
 0.090324
 0.040637
-0.000000

MATRIX A

 -91.000000   101.166549   -13.362249     4.728605    -2.472876     1.625397    -1.261871     1.133164
  -1.193331     1.636614    -1.000000
 -39.001694    30.899183    10.295933    -3.199487     1.611337    -1.042402     0.802801    -0.717749
   0.754018    -1.032887     0.630947
  11.134161   -22.253438     5.554065     7.495047    -2.980513     1.774704    -1.314652     1.151402
  -1.196128     1.629720    -0.994349
  -5.658992     9.932072   -10.764714     1.966642     6.322868    -2.898938     1.949666    -1.630334
   1.653694    -2.228086     1.356122
   3.542337    -5.987251     5.123933    -7.568252     0.724702     5.999549    -3.084008     2.333682
  -2.260062     2.982970    -1.807601
  -2.460938     4.093833    -3.224699     3.667533    -6.341212     0.000000     6.341212    -3.667533
   3.224699    -4.093833     2.460938
   1.807601    -2.982970     2.260062    -2.333682     3.084008    -5.999549    -0.724702     7.568252
  -5.123933     5.987251    -3.542337
  -1.356122     2.228086    -1.653694     1.630334    -1.949666     2.898938    -6.322868    -1.966642
  10.764714    -9.932072     5.658992
   0.994349    -1.629720     1.196128    -1.151402     1.314652    -1.774704     2.980533    -7.495047
  -5.554065    22.253438   -11.134161
  -0.630947     1.032887    -0.754018     0.717749    -0.802801     1.042402    -1.611337     3.199487
 -10.295933   -30.899183    39.001694
   1.000000    -1.636614     1.193331    -1.133164     1.261871    -1.625397     2.472876    -4.728605
  13.362249  -101.166549    91.000000

MATRIX B

 4140.000011  -5702.850841  2105.958935  -811.684653   435.425585  -289.320634   225.849042  -203.426336
  214.586465   -294.537574   180.000000
 2489.506249  -3273.041904   947.964871  -233.795069   109.587772   -68.725505    52.096454   -46.171014
   48.268781    -65.964431    40.273795
 -147.936727    426.493493  -457.448086   217.901747   -56.403667    28.204734   -19.135492    15.967511
  -16.148185     21.716333   -13.211661
   36.288647    -72.911632   151.043493  -206.083900   112.347658   -30.307286    15.986054   -11.728538
```

```
CONTINUED

   11.068239     -14.398352       8.696217
  -15.834157      28.515327     -32.621464      93.738676    -140.759455      82.706389     -23.492179      13.338164
  -11.067148      13.555777      -8.079932
    9.843750     -16.913963      15.428607     -23.917206      78.225379    -125.333333      78.225379     -23.917206
   15.428607     -16.913963       9.843750
   -8.079932      13.555777     -11.067148      13.338164     -23.492179      82.706389    -140.759455      93.738676
  -32.621464      28.515327     -15.834157
    8.696217     -14.398352      11.068239     -11.728538      15.986054     -30.307286     112.347658    -206.083900
  151.043493     -72.911632      36.288647
  -13.211661      21.716333     -16.148185      15.967511     -19.135492      28.204734     -56.403667     217.901747
 -457.448086     426.493493    -147.936727
   40.273795     -65.964431      48.268731     -46.171014      52.096454     -68.725505     109.587772    -233.795069
  947.964871   -3273.041901    2489.506249
  180.000000    -294.537571     214.586465    -203.426336     225.849042    -289.320634     435.425505    -811.684653
 2105.958935   -5702.850341    4140.000011
```

```
     NUMBER OF INTERIOR COLLOCATION POINTS = 10


VECTOR W (WEIGHTING FACTOR)

-0.000000
 0.033336
 0.074726
 0.109543
 0.134633
 0.147762
 0.147762
 0.134633
 0.109543
 0.074726
 0.033336
-0.000000


MATRIX A

   -111.000000     123.325679     -16.153004       5.627928      -2.874307       1.826900      -1.353430       1.136180
     -1.074342       1.168664      -1.630267       1.000000
    -47.636819      37.817153      12.445910      -3.807701       1.872636      -1.171338       0.860712      -0.719235
      0.678247      -0.736636       1.026793      -0.629721
     13.600252     -27.128829       6.874711       8.917478      -3.461866       1.992175      -1.407162       1.150885
     -1.071928       1.155985      -1.605675       0.983973
     -6.915299      12.112547     -13.013984       2.523797       7.338108      -3.248816       2.080943      -1.622278
      1.471866      -1.564350       2.157548      -1.320092
      4.334776      -7.311348       6.200833      -9.006492       1.067255       6.711363      -3.279449       2.307362
     -1.991117       2.061445      -2.808114       1.713487
     -3.022441       5.016907      -3.914507       4.374278      -7.362417       0.304497       6.717074      -3.597581
      2.801828      -2.764989       3.686480      -2.239129
      2.239129      -3.686480       2.764989      -2.801828       3.597581      -6.717074      -0.304497       7.362417
     -4.374278       3.914507      -5.016907       3.022441
     -1.713487       2.808114      -2.061445       1.991117      -2.307362       3.279449      -6.711363      -1.067255
      9.006492      -6.200833       7.311348      -4.334776
      1.320092      -2.157548       1.564350      -1.471866       1.622278      -2.080943       3.248816      -7.338108
     -2.523797      13.013984     -12.112547       6.915289
     -0.983973       1.605675      -1.155985       1.071928      -1.150885       1.407162      -1.992175       3.461866
     -8.917478      -6.874711      27.128829     -13.600252
      0.629721      -1.026793       0.736636      -0.678247       0.719235      -0.860712       1.171338      -1.872636
      3.807701     -12.445910     -37.817153      47.636819
     -1.000000       1.630267      -1.168664       1.074342      -1.136180       1.353430      -1.826900       2.874307
     -5.627928      16.153004    -123.325679     111.000000


MATRIX B

   6160.000015   -8473.084694    3107.134683   -1179.180443     617.804745    -396.985966     295.749334    -249.061440
    235.945013    -256.936930     358.615685    -220.000000
   3699.510811   -4857.957673    1398.726517    -339.710889     155.493744     -94.272313      68.165742     -56.443110
     52.939669     -57.317264      79.769350     -48.904585
```

```
CONTINUED


 -216.164051    623.982171   -666.992155    314.741507     -79.677630     38.517795    -24.898933     19.369392
  -17.514555     18.566723    -25.569652     15.639388
   51.376341   -103.379240    214.703199   -290.809333     156.351739    -40.893336     20.553173    -14.019909
   11.762164    -11.947680     16.110354     -9.807473
  -21.349145     38.500773    -44.223546    127.214139    -188.672008    108.678602    -29.528747     15.572924
  -11.407168     10.750611    -13.975504      8.439070
   12.363791    -21.268173     19.479068    -30.316203      99.022443   -155.568846     94.328839    -26.905100
   15.237059    -12.591791     15.378440     -9.159522
   -9.159522     15.378440    -12.591791     15.237058     -26.905100     94.328839   -155.568846     99.022443
  -30.316203     19.479063    -21.268178     12.363791
    8.439070    -13.975504     10.750611    -11.407168      15.572924    -29.528747    108.678602   -188.672008
  127.214139    -44.223546     38.500773    -21.349145
   -9.807473     16.110354    -11.947680     11.762164     -14.019909     20.553173    -40.893336    156.351739
 -290.809333    214.703199   -103.379240     51.376341
   15.639388    -25.569652     18.566723    -17.514555      19.369392    -24.898933     38.517795    -79.677630
  314.741508   -666.992155    623.982171   -216.164051
  -40.904586     79.769357    -57.317265     52.939669     -56.443110     68.165742    -94.272313    155.493744
 -339.710889   1398.726517  -4857.957673   3699.510811
 -220.000001    358.615686   -256.936931    235.945013    -249.061441    295.749335   -396.985966    617.804745
-1179.180443   3107.134684  -8473.084695   6160.000015
```

APPENDIX C

Solution of a One-Dimensional Problem

The method of orthogonal collocation on finite elements was used to solve a simple one-dimensional problem. The same approach was applied to solve the two-dimensional fin problem. The problem considered is

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} = 12\ x^2 + 4\ x^3 + 1 \qquad (C.1)$$

$y(0) = 1$

$y(1) = 1$

The analytical solution to the differential equation is $y = x^4 + x + 1$. The approach which is due to Carey and Finlayson (1975) was applied here. The domain $0 \leq x \leq 1$ was divided into NE elements as shown below (Figure C.1).

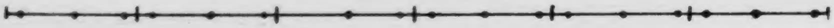


FIGURE C.1 Location of The Collocation Points

For the Figure C.1, NE=6. A new variable u is defined such that

$$u^{\ell} = \frac{x - x_{\ell}}{x_{\ell+1} - x_{\ell}} \qquad \text{where } \ell=1, \ldots, NE \qquad (C.2)$$

Thus u varies from zero to one in each element ℓ. Interior collocation points in each element are the roots to $P_n(u) = 0$ where P_n is a shifted Legendre polynomial in the interval $0 \leq u \leq 1$. Roots to the polynomial are given in Finlayson (1972).

Introducing the variable u and applying OCFE to Equation (C.1) at each interior collocation points, yields,

$$\frac{1}{(\Delta x_\ell)^2} \sum_{i=1}^{NPX} B_{j,i}\, x_i^\ell + \frac{1}{\Delta x_\ell} \sum_{i=1}^{NPX} A_{j,i}\, y_i^\ell = (12\ x^2+4x^3+1)_j^\ell \qquad (C.3)$$

where $\ell = 1, \ldots\ldots NE$

$j = 2, \ldots\ldots N+1$

and $NPX = N+2$

N is the total number of interior collocation points.

At the common point between the elements, continuity of the first derivative is applied, i.e.

$$\frac{1}{\Delta x_\ell} \sum_{i=1}^{NPX} A_{n+2,i}\, y_i^\ell = \frac{1}{\Delta x_{\ell+1}} \sum_{i=1}^{NPX} A_{1,i}\, y_i^{\ell+1} \qquad (C.4)$$

The two boundary conditions become

$$y_1^1 = 1 \text{ and } y_{NPX}^{NE} = 3 \qquad (C.5)$$

Solution of the Equations:

Equation (C.3) together with Equations (C.4) and (C.5) can be written as

$$\bar{\bar{T}}\ \bar{x} = \bar{F} \qquad (C.6)$$

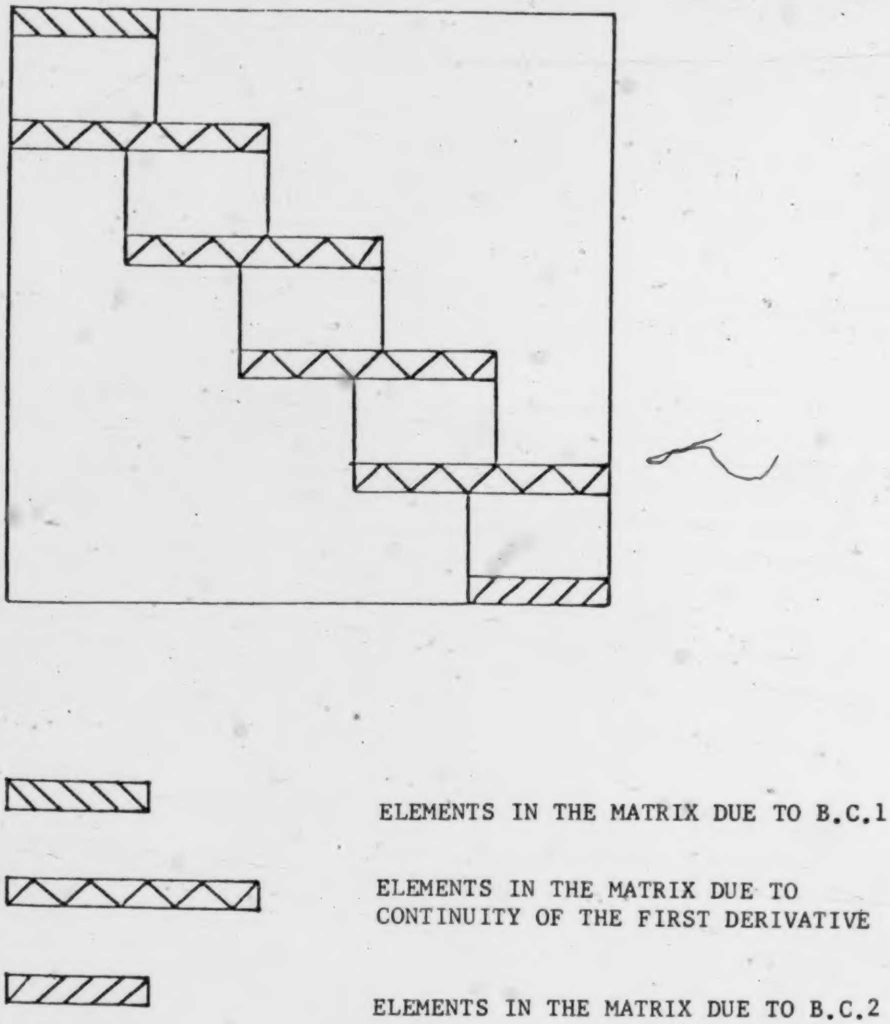
Matrix $\bar{\bar{T}}$ has a block diagonal structure as shown in Figure C.2.

Blocks were stored in a three dimensional array $S(j,i,\ell)$ where

$j = 1 \ldots\ldots N+2$

$i = 1 \ldots\ldots N+2$

$\ell = 1 \ldots\ldots NE$


ELEMENTS IN THE MATRIX DUE TO B.C.1
ELEMENTS IN THE MATRIX DUE TO CONTINUITY OF THE FIRST DERIVATIVE
ELEMENTS IN THE MATRIX DUE TO B.C.2

Clear areas in the blocks represent matrix elements due to the interior collocation points. The size of each block is (N+2) * (N+2)

FIGURE C.2. Block Diagonal Matrix

The scheme is due to Carey and Finlayson and was applied to this problem:

BC1: $S(1,1,1) = 1$

$S(1,i,1) = 0 \quad i=2....N+2$

$F(1) = 1$

BC2: $S(N+2,i,NE) = 0 \quad i = 1.....N+1$

$S(N+2, N+2, NE) = 1$

$F[(N+1)\,NE+1] = 3$

Continuity of the First Derivative:

$\ell = 1....NE-1$

$$S(N+2, i,\ell) = \begin{cases} A(N+2,i) & i = 1....N+1 \\ A(N+2,i) - \dfrac{\Delta x_\ell}{\Delta x_{\ell+1}}\, A(1,1) & i = N+2 \end{cases}$$

$\ell = 2.....NE$

$$S(1,i,\ell) = \begin{cases} S(N+2, N+2, \ell-1) & i = 1 \\ -\dfrac{\Delta x_{\ell-1}}{\Delta x_\ell}\, A(1,i) & i = 2.....N+2 \end{cases}$$

$F[(N+1)(\ell-1) +1] = 0$

Interior Collocation Points:

$\ell = 1.....NE$

$j = 2.....N+1$

$S(j,i,\ell) = B(j,i) + \Delta x_\ell\, A(j,i) \quad i = 1.....N+2$

$F\{(N+1)(\ell-1) + j\} = \Delta x_\ell^2\, [12\, x^2+4x^3 +1]_j^\ell$

Matrix $\bar{\bar{T}}$, which has a block diagonal structure, was converted to a band structure by introducing zeros at the appropriate places as shown in Figure C.3.

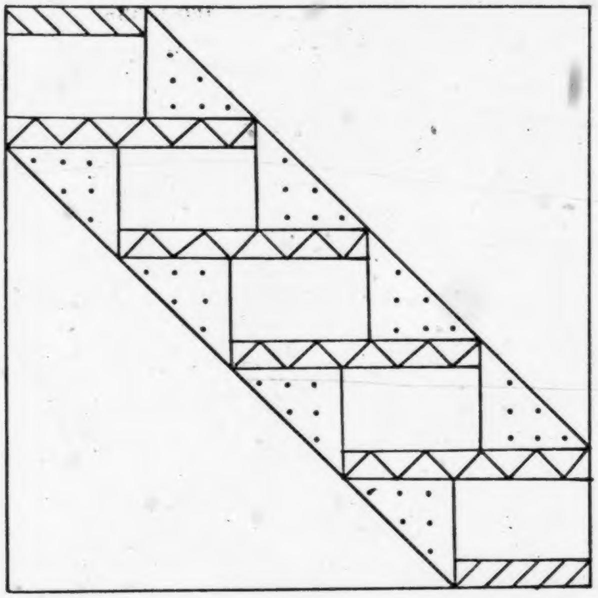


FIGURE C.3. Block Diagonal Matrix as a Band Structured Matrix

Function index in the accompanying computer program introduces zeroes at the appropriate places. Subroutine GELB from the SSP library was used to solve the resulting matrix.

Results:

Table C.1 shows the analytical solution and the solution obtained using OCFE. The numerical solution is in very good agreement with the analytical solution. In order to facilitate the comparison over the interval of (0,1), results for only odd number of collocation points are presented. Various values of NE and NPX were tried. In all the cases considered, excellent agreement with the analytic solution was observed.

Table C.1

Comparison of the Analytical and Numerical Results
For a One-Dimensional Problem.

S.No	X	Analytical Solution	Solution Using OCFE NE=5		
			NPX=3	NPX=5	NPX=7
1	0	1	.99999	.99999	.99999
2	.1	1.1001	1.099151	1.10116	1.10005
3	.2	1.2016	1.199564	1.20172	1.20152
4	.3	1.3081	1.305964	1.30827	1.30804
5	.4	1.4256	1.423069	1.42581	1.42554
6	.5	1.5625	1.56041	1.56272	1.562441
7	.6	1.7296	1.72754	1.72983	1.72954
8	.7	1.9401	1.93879	1.94029	1.94006
9	.8	2.2096	2.20851	2.20976	2.20958
10	.9	2.5561	2.55586	2.55618	2.55609
11	1.0	3.0	3.0	3.0	3.0

```
C=================================================================C
C                                                                 C
C          THE SOLUTION OF A ONE-DIMENSIONAL PROBLEM              C
C                   USING THE METHOD OF                           C
C          ORTHOGONAL COLLOCATION ON FINITE ELEMENTS              C
C                                                                 C
C=================================================================C
C
C
C       THE PROGRAM SOLVES A ONE DIMENSIONAL PROBLEM USING THE OCFE.THE
C       PROGRAM USES SUBROUTINE GELB WHICH IS AVAILABLE IN THE SSP
C       (SCIENTIFIC SUBROUTINE PACKAGE) LIBRARY.
C
C      NE:  TOTAL NUMBER OF ELEMENTS
C      N :  NUMBER OF INTERIOR COLLOCATION POINTS
C      U :  COLLOCATION POINTS
C
C      K1,K2,K3&K4 ARE THE INDICES USED TO INTRODUCE ZEROS AT THE
C      APPROPRIATE PLACES TO CONVERT BLOCK DIAGONAL MATRIX T(STORED AS
C      A THREE DIMENSIONAL MATRIX S )TO A BAND STRUCTURE. MATRIX S IS
C      FINALLY CONVERTED TO  VECTOR M PRIOR TO ENTERING SUBROUTINE GELB
C      IN THE LINE 105.
C
C      A & B ARE THE MATRICES TAKEN FROM APPENDIX 2.
C
C      FORM:  THE VARIABLE FORMAT USED TO READ THE ELEMENTS OF A & B.
C
C
      COMMON / AREA / K1, K2, K3, K4
      DIMENSION S(8, 8, 8), F(60, 1), A(8, 8), D(30, 30), B(8, 8), DX(8)
     &, FORM(8), M(500), U(10), X(50)
      REAL M
      READ (8,120) FORM
      DATA M / 500 * 0. /
C
C      READ THE NUMBER OF INTERIOR COLLOCATION POINTS
C
      READ (19,140) N
      N2 = N + 2
      N1 = N + 1
C
C      READ INPUT
C
      READ (19,150) (U(I), I = 1, N2)
      READ (9,FORM) ((A(I, J), J = 1, N2), I = 1, N2)
      READ (9,FORM) ((B(I, J), J = 1, N2), I = 1, N2)
      READ (5,160) NE
      M1NE = NE - 1
      ANE = 1. / NE
      X(1) = 0
      DO 10 L = 1, NE
      X(L + 1) = X(L) + ANE
      DX(L) = X(L + 1) - X(L)
   10 CONTINUE
      X(NE + 1) = 1.0
C
C      GENERATE ELEMENTS OF MATRIX S AND VECTOR F
C
      S(1, 1, 1) = 1.0
      DO 20 I = 2, N2
      S(1, I, 1) = 0.0
   20 CONTINUE
      F(1, 1) = 1
      DO 40 L = 1, M1NE
      L1 = L + 1
      DO 30 I = 1, N1
      S(N2, I, L) = A(N2, I)
   30 CONTINUE
      S(N2, N2,-L) = A(N2, N2) - DX(L) * A(1, 1) / (DX(L + 1.))
   40 CONTINUE
      DO 60 L = 2, NE
      M1L = L - 1
      S(1, 1, L) = S(N2, N2, M1L)
      DO 50 I = 2, N2
      S(1, I, L) = - DX(M1L) * A(1, I) / DX(L)
   50 CONTINUE
      F((N1 * M1L) + 1, 1) = 0
   60 CONTINUE
      S(N2, N2, NE) = 1.0
      DO 70 I = 1, N1
      S(N2, I, NE) = 0.0
      F(N1 * NE + 1, 1) = 3.
   70 CONTINUE
      DO 100 L = 1, NE
      DO 90 J = 2, N1
      DO 80 I = 1, N2
      S(J, I, L) = B(J, I) + DX(L) * A(J, I)
   80 CONTINUE
      M1L = L - 1
      XC = X(L) + DX(L) * U(J)
      F((N1) * (M1L) + J, 1) = DX(L) * DX(L) * (12 * (XC) ** 2 + 4 * (XC
     &) ** 3. + 1.)
   90 CONTINUE
  100 CONTINUE
      K1 = 3 * N2 * (N2 - 1) / 2
      K2 = (N2 - 1) * (2 * N2 - 1)
      K3 = 2 * N2 - 1
      K4 = K2 * (NE - 2)
C
C     CONVERT S TO M
C
      DO 110 J = 1, N2
      DO 110 I = 1, N2
      DO 110 L = 1, NE
      K = INDEX(J, I, L, N2, NE)
      M(K) = S(J, I, L)
  110 CONTINUE
      N4 = NE * N2 - NE + 1
      CALL GELB(F, M, N4, 1, N1, N1, 1.E - 7, IER)
C
C     WRITE SOLUTION
C
      WRITE (6,130) (F(JK, 1), JK = 1, N4)
      STOP
C
  120 FORMAT (8A4)
  130 FORMAT (F15.6)
  140 FORMAT (I2)
  150 FORMAT (F12.10)
  160 FORMAT (I2)
```

```
C
      END
C
C
      FUNCTION INDEX(J, I, L, N, NE)
      COMMON / AREA / K1, K2, K3, K4
C
C      THE FUNCTION INDEX INTRODUCES ZEROS AT THE APPROPRIATE
C      PLACES TO CONVERT A BLOCK DIAGONAL STRUCTURE TO A BAND STRUCTURE.
C
      IF (L .EQ. 1) GO TO 10
      IF (L .EQ. NE) GO TO 20
      INDEX = K1 + (L - 2) * K2 + (J - 1) * K3 + I - J + 1
      GO TO 30
   10 INDEX = (J - 1) * (J - 2) / 2 + (J - 1) * N + I
      GO TO 30
   20 INDEX = I + (J - 1) * N + J * N - J * (J + 1) / 2 + K1 + K4 - N +
     &1
   30 CONTINUE
      RETURN
      END
```

APPENDIX D

Finite-Difference Formulation of the Finned Tube Problem

The finite-difference formulation of the fin problem is presented. The finite difference method was used to obtain the solution at the corner points of the blocks (Figure 5).

The flow equation is given by

$$\frac{\partial^2 W}{\partial r^2} + \frac{1}{r}\frac{\partial W}{\partial r} + \frac{1}{r^2}\frac{\partial^2 W}{\partial \theta^2} = -1 \tag{D.1}$$

Using a three-points central difference formulation, Equation (D.1) becomes

$$\left[\frac{W_{i,j+1} + W_{i,j-1} - 2W_{i,j}}{\Delta r_i^2}\right] + \frac{1}{r_i}\left[\frac{W_{i,j+1} - W_{i,j-1}}{2\Delta r}\right] + \frac{1}{r_i^2}\left[\frac{W_{i+1,j} + W_{i-1,j} - 2W_{i,j}}{\Delta\theta^2}\right] = -1 \tag{D.2}$$

Rearranging the above equation, one obtains

$$W_{i,j} = \frac{1}{2(Z^2+1)}\left[Z\left(Z + \frac{\Delta\theta}{2}\right) W_{i,j+1} + Z\left(Z - \frac{\Delta\theta}{2}\right) W_{i,j-1} + W_{i+1,j} + W_{i-1,j} + r_i^2\,\Delta\theta^2\right] \tag{D.3}$$

where $Z = \frac{r\,\Delta\theta}{\Delta r}$

Equation (D.3) was used to evaluate the velocity at the corner points (Full circles in Figure 5) of each block.

APPENDIX E

Pressure and Velocity Results from Continuity Equatioon

A value of 10.99 in the velocity columns indicates that the velocity at that collocation point was not evaluated.

TABLE E.1

CONFIGURATION ONE: PRESSURE AND VELOCITY RESULTS FOR VISCOSITY=100 CP AND N=2

INITIAL VISCOSITY OF THE OIL= 100.000 CP
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT= 4

COLLOCATION POINT	PRESSURE (PSI)	VELOCITY IN X DIRECTION CM/S	VELOCITY IN Y DIRECTION CM/S
1	1.543411746	-0.000000000	10.990000000
2	1.215405415	0.000000000	10.990000000
3	1.071168041	-0.000000000	10.990000000
4	1.005110531	-0.000000000	10.990000000
5	1.534741043	10.990000000	-0.000000000
6	1.495667005	0.000002090	0.000001686
7	1.196111658	0.000000857	0.000002462
8	1.111316193	10.990000000	0.000001344
9	1.062034594	0.000000409	0.000000951
10	1.000000000	0.000000229	0.000000175
11	0.996439828	10.990000000	0.0
12	1.090427370	10.990000000	0.000000000
13	1.081496671	0.000004015	0.000000388
14	1.009025892	0.000002035	0.000000647
15	0.984854718	10.990000000	0.000000448
16	0.968384632	0.000001070	0.000000319
17	0.947419607	0.000000602	0.000000060
18	0.946189996	10.990000000	0.000000000
19	0.920299179	0.000003335	10.990000000
20	0.920299179	0.000002041	10.990000000
21	0.920299179	0.000001145	10.990000000
22	0.920299179	0.000000647	10.990000000
23	0.750170988	10.990000000	0.000000000
24	0.759101687	0.000004015	-0.000000388
25	0.831572466	0.000002035	-0.000000647
26	0.855743640	10.990000000	-0.000000448
27	0.872213726	0.000001070	-0.000000319
28	0.893178751	0.000000602	-0.000000060
29	0.894408362	10.990000000	-0.000000000
30	0.305857315	10.990000000	0.000000000
31	0.344931353	0.000002090	-0.000001686
32	0.644486700	0.000000857	-0.000002462
33	0.729282165	10.990000000	-0.000001344
34	0.778563765	0.000000409	-0.000000951
35	0.840598358	0.000000229	-0.000000175
36	0.844158530	10.990000000	0.0
37	0.297186613	-0.000000000	10.990000000
38	0.625192943	-0.000000000	10.990000000
39	0.769430317	-0.000000000	10.990000000
40	0.835487827	0.0	10.990000000

TABLE E.2

CONFIGURATION ONE: PRESSURE AND VELOCITY RESULTS FOR VISCOSITY=10,000 CP AND N=2

```
INITIAL VISCOSITY OF THE OIL= 10000.000 CP
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT=  4


COLLOCATION  PRESSURE           VELOCITY IN       VELOCITY IN
   POINT       (PSI)            X DIRECTION       Y DIRECTION
                                   CM/S              CM/S

      1      55.341174562      -0.000000000      10.990000000
      2      22.540541474       0.000000000      10.990000000
      3       8.116804134       0.0              10.990000000
      4       1.511053071      -0.000000000      10.990000000
      5      54.474104294      10.990000000       0.000000000
      6      50.566700532       0.000002090       0.000001686
      7      20.611165769       0.000000857       0.000002462
      8      12.131619278      10.990000000       0.000001344
      9       7.203459352       0.000000409       0.000000951
     10       1.000000000       0.000000229       0.000000175
     11       0.643982802      10.990000000       0.000000000
     12      10.042736982      10.990000000       0.000000000
     13       9.149667087       0.000004015       0.000000388
     14       1.902589223       0.000002035       0.000000647
     15      -0.514528197      10.990000000       0.000000448
     16      -2.161536751       0.000001070       0.000000319
     17      -4.258039330       0.000000602       0.000000060
     18      -4.381000358      10.990000000      -0.000000000
     19      -6.970082094       0.000003335      10.990000000
     20      -6.970082094       0.000002041      10.990000000
     21      -6.970082094       0.000001145      10.990000000
     22      -6.970082094       0.000000647      10.990000000
     23     -23.982901169      10.990000000      -0.000000000
     24     -23.089831275       0.000004015      -0.000000388
     25     -15.842753411       0.000002035      -0.000000647
     26     -13.425635991      10.990000000      -0.000000448
     27     -11.778627437       0.000001070      -0.000000319
     28      -9.682124858       0.000000602      -0.000000060
     29      -9.559163829      10.990000000       0.0
     30     -68.414268481      10.990000000      -0.000000000
     31     -64.506864720       0.000002090      -0.000001686
     32     -34.551329956       0.000000857      -0.000002462
     33     -26.071783466      10.990000000      -0.000001344
     34     -21.143623539       0.000000409      -0.000000951
     35     -14.940164188       0.000000229      -0.000000175
     36     -14.584146990      10.990000000       0.0
     37     -69.281338750       0.0              10.990000000
     38     -36.480705661      -0.000000000      10.990000000
     39     -22.056968321      -0.000000000      10.990000000
     40     -15.451217259      -0.000000000      10.990000000
```

TABLE E.3

CONFIGURATION ONE: PRESSURE AND VELOCITY RESULTS FOR VISCOSITY=100 CP AND N=3

INITIAL VISCOSITY OF THE OIL= 100.000 CP
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT= 9

COLLOCATION POINT	PRESSURE (PSI)	VELOCITY IN X DIRECTION CM/S	VELOCITY IN Y DIRECTION CM/S
1	1.828773953	-0.000000000	10.990000000
2	1.395770126	0.000000000	10.990000000
3	1.156293268	-0.000000000	10.990000000
4	1.093897308	-0.000000000	10.990000000
5	1.026436607	0.000000000	10.990000000
6	1.001309415	-0.000000000	10.990000000
7	1.826174900	10.990000000	-0.000000000
8	1.783944216	0.000003598	0.000003373
9	1.381502060	0.000001174	0.000004810
10	1.152174607	0.000000353	0.000001360
11	1.121433160	10.990000000	0.000001474
12	1.090773736	0.000000267	0.000001199
13	1.024689370	0.000000151	0.000000533
14	1.000000000	0.000000113	0.000000112
15	0.998710335	10.990000000	0.000000000
16	1.345640597	10.990000000	0.000000000
17	1.334006465	0.000005789	0.000000946
18	1.203668649	0.000002820	0.000001860
19	1.083631182	0.000001307	0.000001115
20	1.060035134	10.990000000	0.000000955
21	1.039920776	0.000000960	0.000000797
22	0.994634706	0.000000583	0.000000377
23	0.977193298	0.000000445	0.000000077
24	0.976306574	10.990000000	-0.000000000
25	0.995908837	10.990000000	0.000000000
26	0.995131576	0.000003179	0.000000067
27	0.981726407	0.000002717	0.000000254
28	0.959401085	0.000001736	0.000000271
29	0.953659550	10.990000000	0.000000224
30	0.948916384	0.000001276	0.000000189
31	0.938014976	0.000000805	0.000000092
32	0.933729126	0.000000621	0.000000019
33	0.933511264	10.990000000	-0.000000000
34	0.919316895	0.000003576	10.990000000
35	0.919316689	0.000002726	10.990000000
36	0.919316243	0.000001739	10.990000000
37	0.919315898	0.000001291	10.990000000
38	0.919315274	0.000000816	10.990000000

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39        0.919314863      0.000000629     10.990000000
40        0.842724974     10.990000000     -0.000000000
41        0.843502213      0.000003179     -0.000000067
42        0.856906969      0.000002717     -0.000000254
43        0.879231393      0.000001736     -0.000000271
44        0.884572592     10.990000000     -0.000000224
45        0.889715394      0.000001276     -0.000000189
46        0.900615535      0.000000805     -0.000000092
47        0.904500598      0.000000621     -0.000000019
48        0.905118409     10.990000000      0.0
49        0.492993197     10.990000000      0.000000000
50        0.504627306      0.000005789     -0.000000946
51        0.634964686      0.000002820     -0.000001860
52        0.755001169      0.000001307     -0.000001115
53        0.778596830     10.990000000     -0.000000955
54        0.798710760      0.000000960     -0.000000797
55        0.843995013      0.000000583     -0.000000377
56        0.861434767      0.000000445     -0.000000077
57        0.862321354     10.990000000      0.0
58        0.012458879     10.990000000      0.000000000
59        0.054689537      0.000003598     -0.000003373
60        0.457131234      0.000001174     -0.000004810
61        0.686457602      0.000000353     -0.000001359
62        0.717198598     10.990000000     -0.000001474
63        0.747857567      0.000000267     -0.000001199
64        0.813939173      0.000000151     -0.000000533
65        0.838624250      0.000000113     -0.000000112
66        0.839913476     10.990000000      0.0
67        0.009859799      0.000000000     10.990000000
68        0.442863164     -0.000000000     10.990000000
69        0.682338930      0.0             10.990000000
70        0.744733978      0.000000000     10.990000000
71        0.812191825      0.0             10.990000000
72        0.837314422      0.000000000     10.990000000
```

TABLE E.4

CONFIGURATION ONE: PRESSURE AND VELOCITY RESULTS FOR VISCOSITY=10,000 CP AND N=3

```
INITIAL VISCOSITY OF THE OIL= 10000.000 CP
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT=  9


COLLOCATION   PRESSURE          VELOCITY IN       VELOCITY IN
   POINT        (PSI)           X DIRECTION       Y DIRECTION
                                   CM/S              CM/S

      1      83.876459984      -0.000000000      10.990000000
      2      40.576115792      -0.000000000      10.990000000
      3      16.628525291       0.000000000      10.990000000
      4      10.389012177      -0.000000000      10.990000000
      5       3.643225378      -0.000000000      10.990000000
      6       1.130987975      -0.000000000      10.990000000
      7      83.616551825      10.990000000       0.000000000
      8      79.393465553       0.000003598       0.000003373
      9      39.149308027       0.000001174       0.000004810
     10      16.216656916       0.000000353       0.000001359
     11      13.142553144      10.990000000       0.000001474
     12      10.076651862       0.000000267       0.000001199
     13       3.468488009       0.000000151       0.000000533
     14       1.000000000       0.000000113       0.000000113
     15       0.871080016      10.990000000       0.000000006
     16      35.563108625      10.990000000      -0.000000000
     17      34.399697140       0.000005789       0.000000946
     18      21.365948412       0.000002820       0.000001860
     19       9.362279245       0.000001307       0.000001115
     20       7.002706139      10.990000000       0.000000955
     21       4.991306181       0.000000960       0.000000797
     22       0.462861981       0.000000583       0.000000377
     23      -1.281119206       0.000000445       0.000000077
     24      -1.369777914      10.990000000      -0.000000000
     25       0.589908505      10.990000000       0.0
     26       0.512183752       0.000003179       0.000000067
     27      -0.828308604       0.000002717       0.000000254
     28      -3.060786558       0.000001736       0.000000271
     29      -3.634919391      10.990000000       0.000000224
     30      -4.109213282       0.000001276       0.000000189
     31      -5.199273790       0.000000805       0.000000092
     32      -5.627809161       0.000000621       0.000000019
     33      -5.649592209      10.990000000       0.0
     34      -7.069292227       0.000003576      10.990000000
     35      -7.069290645       0.000002726      10.990000000
     36      -7.069287217       0.000001739      10.990000000
     37      -7.069284562       0.000001291      10.990000000
     38      -7.069279766       0.000000816      10.990000000
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 39     -7.069276602      0.000000629     10.990000000
 40    -14.728493119     10.990000000     -0.000000000
 41    -14.650768197      0.000003179     -0.000000067
 42    -13.310272663      0.000002717     -0.000000254
 43    -11.077787808      0.000001736     -0.000000271
 44    -10.503652388     10.990000000     -0.000000224
 45    -10.029355697      0.000001276     -0.000000189
 46     -8.939285451      0.000000805     -0.000000092
 47     -8.510744027      0.000000621     -0.000000019
 48     -8.488960588     10.990000000      0.0
 49    -49.701693115     10.990000000     -0.000000000
 50    -48.538281452      0.000005789     -0.000000946
 51    -35.504529371      0.000002820     -0.000001860
 52    -23.500852634      0.000001307     -0.000001115
 53    -21.141276550     10.990000000     -0.000000955
 54    -19.129873300      0.000000960     -0.000000797
 55    -14.601415134      0.000000583     -0.000000377
 56    -12.857421228      0.000000445     -0.000000077
 57    -12.768761461     10.990000000      0.000000000
 58    -97.755136192     10.990000000     -0.000000000
 59    -93.532069728      0.000003598     -0.000003373
 60    -53.287888666      0.000001174     -0.000004810
 61    -30.355229219      0.000000353     -0.000001359
 62    -27.281121972     10.990000000     -0.000001474
 63    -24.215217190      0.000000267     -0.000001199
 64    -17.607032114      0.000000151     -0.000000533
 65    -15.138511095      0.000000113     -0.000000112
 66    -15.009587734     10.990000000     -0.000000000
 67    -98.015044150      0.0             10.990000000
 68    -54.714696407      0.0             10.990000000
 69    -30.767097506     -0.000000000     10.990000000
 70    -24.527577373      0.000000000     10.990000000
 71    -17.781768628      0.000000000     10.990000000
 72    -15.269495894     -0.000000000     10.990000000
```

TABLE E.5

CONFIGURATION ONE: PRESSURE AND VELOCITY RESULTS FOR VISCOSITY=100 CP AND N=4

INITIAL VISCOSITY OF THE OIL= 100.000 CP
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT= 16

COLLOCATION POINT	PRESSURE (PSI)	VELOCITY IN X DIRECTION CM/S	VELOCITY IN Y DIRECTION CM/S
1	2.052622302	-0.000000000	10.990000000
2	1.576227614	-0.000000000	10.990000000
3	1.257984014	0.000000000	10.990000000
4	1.151082134	0.000000000	10.990000000
5	1.104371144	0.000000000	10.990000000
6	1.049439716	-0.000000000	10.990000000
7	1.011155078	0.000000000	10.990000000
8	1.000483946	-0.000000000	10.990000000
9	2.051661239	10.990000000	0.000000000
10	2.008709052	0.000005675	0.000005539
11	1.563697236	0.000001665	0.000007865
12	1.255522323	0.000000347	0.000002013
13	1.149150717	0.000000266	0.000002104
14	1.122802072	10.990000000	0.000001471
15	1.103206766	0.000000164	0.000001297
16	1.048643602	0.000000112	0.000000787
17	1.010604857	0.000000078	0.000000337
18	1.000000000	0.000000068	0.000000067
19	0.999522883	10.990000000	0.0
20	1.554434892	10.990000000	0.000000000
21	1.542870006	0.000008513	0.000001529
22	1.394136275	0.000003905	0.000003260
23	1.204814370	0.000001496	0.000002071
24	1.114159180	0.000000985	0.000001409
25	1.095285547	10.990000000	0.000001247
26	1.078635169	0.000000741	0.000001105
27	1.031894936	0.000000507	0.000000678
28	0.998990386	0.000000353	0.000000292
29	0.989777627	0.000000311	0.000000059
30	0.989362356	10.990000000	-0.000000000
31	1.167761337	10.990000000	0.000000000
32	1.166332990	0.000003342	0.000000202
33	1.137555013	0.000003363	0.000000826
34	1.070708111	0.000002204	0.000000963
35	1.026284472	0.000001484	0.000000696
36	1.016831511	10.990000000	0.000000643
37	1.008160147	0.000001228	0.000000581
38	0.982947021	0.000000867	0.000000374

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CONTINUED

39        0.964635580     0.000000613     0.000000164
40        0.959454270     0.000000542     0.000000033
41        0.959217038    10.990000000     0.000000000
42        0.974780952    10.990000000     0.000000000
43        0.974065324     0.000004021     0.000000095
44        0.964775940     0.000003213     0.000000208
45        0.951346964     0.000002255     0.000000176
46        0.942505164     0.000001633     0.000000153
47        0.940443575    10.990000000     0.000000136
48        0.938607231     0.000001357     0.000000123
49        0.933222639     0.000000978     0.000000081
50        0.929251928     0.000000699     0.000000036
51        0.928121373     0.000000620     0.000000007
52        0.928069962    10.990000000    -0.000000000
53        0.919306346     0.000003565    10.990000000
54        0.919306346     0.000003198    10.990000000
55        0.919306346     0.000002270    10.990000000
56        0.919306346     0.000001637    10.990000000
57        0.919306346     0.000001366    10.990000000
58        0.919306346     0.000000984    10.990000000
59        0.919306346     0.000000703    10.990000000
60        0.919306346     0.000000623    10.990000000
61        0.863831740    10.990000000     0.000000000
62        0.864547368     0.000004021    -0.000000095
63        0.873836752     0.000003213    -0.000000208
64        0.887265728     0.000002255    -0.000000176
65        0.896107528     0.000001633    -0.000000153
66        0.898169117    10.990000000    -0.000000136
67        0.900005461     0.000001357    -0.000000123
68        0.905390053     0.000000978    -0.000000081
69        0.909360763     0.000000699    -0.000000036
70        0.910491319     0.000000620    -0.000000007
71        0.910542730    10.990000000    -0.000000000
72        0.670851355    10.990000000    -0.000000000
73        0.672279701     0.000003342    -0.000000202
74        0.701057679     0.000003363    -0.000000826
75        0.767904580     0.000002204    -0.000000963
76        0.812328220     0.000001484    -0.000000696
77        0.821781181    10.990000000    -0.000000643
78        0.830452545     0.000001228    -0.000000581
79        0.855665670     0.000000867    -0.000000374
80        0.873977112     0.000000613    -0.000000164
81        0.879158422     0.000000542    -0.000000033
82        0.879395654    10.990000000     0.000000000
83        0.284177799    10.990000000    -0.000000000
84        0.295742686     0.000008513    -0.000001529
85        0.444476417     0.000003905    -0.000003260
86        0.633798322     0.000001496    -0.000002071
87        0.724453512     0.000000985    -0.000001409
88        0.743327145    10.990000000    -0.000001247
89        0.759977523     0.000000741    -0.000001105
90        0.806717755     0.000000507    -0.000000678
91        0.839622306     0.000000353    -0.000000292
92        0.848835065     0.000000311    -0.000000059
93        0.849250336    10.990000000     0.0
94       -0.213048547    10.990000000     0.000000000
95       -0.170096360     0.000005675    -0.000005539
96        0.274915456     0.000001665    -0.000007865
97        0.583090369     0.000000347    -0.000002013
98        0.689461975     0.000000266    -0.000002104
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 99       0.715810620     10.990000000     -0.000001471
100       0.735405926      0.000000164     -0.000001297
101       0.789969090      0.000000112     -0.000000787
102       0.828007835      0.000000078     -0.000000337
103       0.838612692      0.000000068     -0.000000067
104       0.839089809     10.990000000      0.0
105      -0.214009610      0.0             10.990000000
106       0.262385077      0.000000000     10.990000000
107       0.580628677     -0.000000000     10.990000000
108       0.687530558     -0.000000000     10.990000000
109       0.734241548     -0.000000000     10.990000000
110       0.789172976      0.000000000     10.990000000
111       0.827457614      0.0             10.990000000
112       0.838128745     -0.000000000     10.990000000
```

TABLE E.6

CONFIGURATION ONE: PRESSURE AND VELOCITY RESULTS FOR VISCOSITY=10,000 CP AND N=4

INITIAL VISCOSITY OF THE OIL= 10000.000 CP
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT= 16

COLLOCATION POINT	PRESSURE (PSI)	VELOCITY IN X DIRECTION CM/S	VELOCITY IN Y DIRECTION CM/S
1	106.262230183	0.000000000	10.990000000
2	58.622761447	0.000000000	10.990000000
3	26.798401447	0.000000000	10.990000000
4	16.108213384	0.000000000	10.990000000
5	11.437114364	0.000000000	10.990000000
6	5.943971577	0.000000000	10.990000000
7	2.115507831	0.000000000	10.990000000
8	1.048394643	-0.000000000	10.990000000
9	106.166123865	10.990000000	-0.000000000
10	101.870905227	0.000005675	0.000005539
11	57.369723610	0.000001665	0.000007865
12	26.552232257	0.000000347	0.000002013
13	15.915071659	0.000000266	0.000002104
14	13.280207193	10.990000000	0.000001471
15	11.320676552	0.000000164	0.000001297
16	5.864360170	0.000000112	0.000000787
17	2.060485724	0.000000078	0.000000337
18	1.000000000	0.000000068	0.000000067
19	0.952288325	10.990000000	0.000000000
20	56.443489234	10.990000000	0.000000000
21	55.287000559	0.000008513	0.000001529
22	40.413627464	0.000003905	0.000003260
23	21.481436958	0.000001496	0.000002071
24	12.415918000	0.000000985	0.000001409
25	10.528554669	10.990000000	0.000001247
26	8.863516891	0.000000741	0.000001105
27	4.189493644	0.000000507	0.000000678
28	0.899038611	0.000000353	0.000000292
29	-0.022237327	0.000000311	0.000000059
30	-0.063764382	10.990000000	-0.000000000
31	17.776133684	10.990000000	-0.000000000
32	17.633295039	0.000003342	0.000000202
33	14.755501308	0.000003363	0.000000826
34	8.070811137	0.000002204	0.000000963
35	3.628447157	0.000001484	0.000000696
36	2.683151075	10.990000000	0.000000643
37	1.816014659	0.000001228	0.000000581
38	-0.705297857	0.000000867	0.000000374

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 39     -2.536442046      0.000000613     0.000000164
 40     -3.054573048      0.000000542     0.000000033
 41     -3.078296186     10.990000000     0.0
 42     -1.521904804     10.990000000     0.000000000
 43     -1.593467621      0.000004021     0.000000095
 44     -2.522406045      0.000003213     0.000000208
 45     -3.865303603      0.000002255     0.000000176
 46     -4.749483618      0.000001633     0.000000153
 47     -4.955642527     10.990000000     0.000000136
 48     -5.139276913      0.000001357     0.000000123
 49     -5.677736141      0.000000978     0.000000081
 50     -6.074807154      0.000000699     0.000000036
 51     -6.187862728      0.000000620     0.000000007
 52     -6.193003824     10.990000000     0.000000000
 53     -7.069365408      0.000003565    10.990000000
 54     -7.069365408      0.000003198    10.990000000
 55     -7.069365408      0.000002270    10.990000000
 56     -7.069365408      0.000001637    10.990000000
 57     -7.069365408      0.000001366    10.990000000
 58     -7.069365408      0.000000984    10.990000000
 59     -7.069365408      0.000000703    10.990000000
 60     -7.069365408      0.000000623    10.990000000
 61    -12.616826012     10.990000000     0.000000000
 62    -12.545263195      0.000004021    -0.000000095
 63    -11.616324771      0.000003213    -0.000000208
 64    -10.273427213      0.000002255    -0.000000176
 65     -9.389247198      0.000001633    -0.000000153
 66     -9.183088289     10.990000000    -0.000000136
 67     -8.999453903      0.000001357    -0.000000123
 68     -8.460994675      0.000000978    -0.000000081
 69     -8.063923662      0.000000699    -0.000000036
 70     -7.950868088      0.000000620    -0.000000007
 71     -7.945726992     10.990000000     0.0
 72    -31.914864500     10.990000000     0.000000000
 73    -31.772029855      0.000003342    -0.000000202
 74    -28.894232124      0.000003363    -0.000000826
 75    -22.209541953      0.000002204    -0.000000963
 76    -17.767178013      0.000001484    -0.000000696
 77    -16.821881891     10.990000000    -0.000000643
 78    -15.954745474      0.000001228    -0.000000581
 79    -13.433432959      0.000000867    -0.000000374
 80    -11.602288769      0.000000613    -0.000000164
 81    -11.084157768      0.000000542    -0.000000033
 82    -11.060434630     10.990000000     0.000000000
 83    -70.582220050     10.990000000    -0.000000000
 84    -69.425731375      0.000008513    -0.000001529
 85    -54.552358280      0.000003905    -0.000003260
 86    -35.620167814      0.000001496    -0.000002071
 87    -26.554648816      0.000000985    -0.000001409
 88    -24.667285485     10.990000000    -0.000001247
 89    -23.002247707      0.000000741    -0.000001105
 90    -18.328224459      0.000000507    -0.000000678
 91    -15.037769427      0.000000353    -0.000000292
 92    -14.116493489      0.000000311    -0.000000059
 93    -14.074966434     10.990000000    -0.000000000
 94   -120.304654681     10.990000000    -0.000000000
 95   -116.009636043      0.000005675    -0.000005539
 96    -71.508454426      0.000001665    -0.000007865
 97    -40.690963073      0.000000347    -0.000002013
 98    -30.053802475      0.000000266    -0.000002104
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 99    -27.418938009      10.990000000     -0.000001471
100    -25.459407368       0.000000164     -0.000001297
101    -20.003090986       0.000000112     -0.000000787
102    -16.199216540       0.000000078     -0.000000337
103    -15.138730816       0.000000068     -0.000000067
104    -15.091019141      10.990000000      0.000000000
105   -120.400960999       0.000000000     10.990000000
106    -72.761492263       0.000000000     10.990000000
107    -40.937132263       0.000000000     10.990000000
108    -30.246944199       0.000000000     10.990000000
109    -25.575845179      -0.000000000     10.990000000
110    -20.082702393       0.0             10.990000000
111    -16.254238647       0.000000000     10.990000000
112    -15.187125459       0.0             10.990000000
```

TABLE E.7

CONFIGURATION ONE: PRESSURE AND VELOCITY RESULTS FOR VISCOSITY=100 CP AND N=5

INITIAL VISCOSITY OF THE OIL= 100.000 CP
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT= 25

COLLOCATION POINT	PRESSURE (PSI)	VELOCITY IN X DIRECTION CM/S	VELOCITY IN Y DIRECTION CM/S
1	2.234607128	-0.000000000	10.990000000
2	1.737433350	0.000000000	10.990000000
3	1.376337514	-0.000000000	10.990000000
4	1.216296492	-0.000000000	10.990000000
5	1.136850047	-0.000000000	10.990000000
6	1.110997583	-0.000000000	10.990000000
7	1.067341590	0.000000000	10.990000000
8	1.026412695	-0.000000000	10.990000000
9	1.005394978	0.000000000	10.990000000
10	1.000220212	-0.000000000	10.990000000
11	2.234165861	10.990000000	-0.000000000
12	2.190994862	0.000008310	0.000008218
13	1.725676028	0.000002306	0.000011613
14	1.374494719	0.000000389	0.000002910
15	1.215013063	0.000000262	0.000002984
16	1.136410532	0.000000097	0.000001210
17	1.124150168	10.990000000	0.000001501
18	1.110398502	0.000000124	0.000001373
19	1.066915711	0.000000089	0.000000966
20	1.026116075	0.000000062	0.000000544
21	1.005159837	0.000000049	0.000000231
22	1.000000000	0.000000046	0.000000046
23	0.999778939	10.990000000	0.000000000
24	1.726776296	10.990000000	0.000000000
25	1.715459344	0.000012065	0.000002214
26	1.560109222	0.000005339	0.000004887
27	1.331811033	0.000001817	0.000003230
28	1.189845828	0.000001019	0.000002081
29	1.123187673	0.000000624	0.000001474
30	1.109674172	10.990000000	0.000001357
31	1.097194358	0.000000567	0.000001251
32	1.057230470	0.000000422	0.000000894
33	1.019322084	0.000000297	0.000000507
34	0.999763864	0.000000236	0.000000216
35	0.994943017	0.000000222	0.000000043
36	0.994737782	10.990000000	0.000000000
37	1.326227252	10.990000000	-0.000000000
38	1.324835028	0.000003895	0.000000294

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CONTINUED

   39      1.292556261      0.000004206      0.000001358
   40      1.199988026      0.000002731      0.000001769
   41      1.114299050      0.000001663      0.000001317
   42      1.070058200      0.000001244      0.000001047
   43      1.060406743     10.990000000      0.000000965
   44      1.051507648      0.000001062      0.000000895
   45      1.022612394      0.000000815      0.000000653
   46      0.994660298      0.000000586      0.000000377
   47      0.980066789      0.000000469      0.000000162
   48      0.976455723      0.000000441      0.000000032
   49      0.976301766     10.990000000      0.0
   50      1.096821423     10.990000000      0.000000000
   51      1.096040164      0.000004532      0.000000157
   52      1.082352697      0.000003593      0.000000517
   53      1.048646139      0.000002736      0.000000663
   54      1.013319300      0.000001960      0.000000595
   55      0.993121893      0.000001515      0.000000479
   56      0.988671966     10.990000000      0.000000451
   57      0.984485416      0.000001339      0.000000423
   58      0.970635880      0.000001048      0.000000317
   59      0.956958608      0.000000766      0.000000186
   60      0.949735906      0.000000618      0.000000080
   61      0.947941227      0.000000581      0.000000016
   62      0.947864928     10.990000000      0.0
   63      0.951092704     10.990000000     -0.000000000
   64      0.951268170      0.000003202     -0.000000031
   65      0.951620697      0.000003382      0.000000041
   66      0.945670817      0.000002756      0.000000143
   67      0.938572534      0.000002009      0.000000111
   68      0.934560825      0.000001591      0.000000102
   69      0.933622680     10.990000000      0.000000092
   70      0.932766954      0.000001403      0.000000086
   71      0.929935330      0.000001108      0.000000065
   72      0.927118607      0.000000814      0.000000038
   73      0.925625095      0.000000658      0.000000017
   74      0.925253672      0.000000619      0.000000003
   75      0.925237764     10.990000000      0.0
   76      0.919314279      0.000003623     10.990000000
   77      0.919314096      0.000003374     10.990000000
   78      0.919313838      0.000002745     10.990000000
   79      0.919313490      0.000002013     10.990000000
   80      0.919313277      0.000001591     10.990000000
   81      0.919313086      0.000001407     10.990000000
   82      0.919312565      0.000001110     10.990000000
   83      0.919311957      0.000000816     10.990000000
   84      0.919311559      0.000000660     10.990000000
   85      0.919311548      0.000000621     10.990000000
   86      0.887535858     10.990000000      0.000000000
   87      0.887360300      0.000003202      0.000000031
   88      0.887007407      0.000003382     -0.000000041
   89      0.892956771      0.000002756     -0.000000143
   90      0.900054357      0.000002009     -0.000000111
   91      0.904065639      0.000001591     -0.000000102
   92      0.905003683     10.990000000     -0.000000092
   93      0.905859127      0.000001403     -0.000000086
   94      0.908689709      0.000001108     -0.000000065
   95      0.911505210      0.000000814     -0.000000038
   96      0.912997921      0.000000658     -0.000000017
   97      0.913369357      0.000000619     -0.000000003
   98      0.913385344     10.990000000      0.0
```

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CONTINUED

  99     0.741806887    10.990000000     0.000000000
 100     0.742588053     0.000004532    -0.000000157
 101     0.756275154     0.000003593    -0.000000517
 102     0.789981189     0.000002736    -0.000000663
 103     0.825307313     0.000001960    -0.000000595
 104     0.845504271     0.000001515    -0.000000479
 105     0.849954089    10.990000000    -0.000000451
 106     0.854140350     0.000001339    -0.000000423
 107     0.867588797     0.000001048    -0.000000317
 108     0.881664743     0.000000766    -0.000000186
 109     0.888886499     0.000000618    -0.000000080
 110     0.890681082     0.000000581    -0.000000016
 111     0.890757450    10.990000000    -0.000000000
 112     0.512400976    10.990000000     0.000000000
 113     0.513793108     0.000003895    -0.000000294
 114     0.546071503     0.000004206    -0.000001358
 115     0.638639194     0.000002731    -0.000001769
 116     0.724327403     0.000001663    -0.000001317
 117     0.768567732     0.000001244    -0.000001047
 118     0.778219053    10.990000000    -0.000000965
 119     0.787117828     0.000001062    -0.000000895
 120     0.816011833     0.000000815    -0.000000653
 121     0.843962176     0.000000586    -0.000000377
 122     0.858554142     0.000000469    -0.000000162
 123     0.862164920     0.000000441    -0.000000032
 124     0.862318942    10.990000000     0.000000000
 125     0.111851979    10.990000000     0.000000000
 126     0.123168839     0.000012065    -0.000002214
 127     0.278518585     0.000005339    -0.000004887
 128     0.506816208     0.000001817    -0.000003230
 129     0.648780591     0.000001019    -0.000002081
 130     0.715438145     0.000000624    -0.000001474
 131     0.728951484    10.990000000    -0.000001357
 132     0.741430946     0.000000567    -0.000001251
 133     0.781393369     0.000000422    -0.000000894
 134     0.819299371     0.000000297    -0.000000507
 135     0.838854326     0.000000236    -0.000000216
 136     0.843673269     0.000000222    -0.000000043
 137     0.843878438    10.990000000    -0.000000000
 138    -0.395537344    10.990000000     0.000000000
 139    -0.352366438     0.000008310    -0.000008218
 140     0.112952019     0.000002306    -0.000011613
 141     0.464132755     0.000000389    -0.000002910
 142     0.623613570     0.000000262    -0.000002984
 143     0.702215471     0.000000097    -0.000001210
 144     0.714475660    10.990000000    -0.000001501
 145     0.728226996     0.000000124    -0.000001373
 146     0.771708184     0.000000089    -0.000000966
 147     0.812505230     0.000000062    -0.000000544
 148     0.833456586     0.000000049    -0.000000231
 149     0.838610392     0.000000046    -0.000000046
 150     0.838830961    10.990000000     0.000000000
 151    -0.395978617    -0.000000000    10.990000000
 152     0.101194783     0.0            10.990000000
 153     0.462290047    -0.000000000    10.990000000
 154     0.622330226     0.0            10.990000000
 155     0.701776040     0.0            10.990000000
 156     0.727628001     0.0            10.990000000
 157     0.771282381     0.0            10.990000000
 158     0.812208681     0.0            10.990000000
 159     0.833221384     0.000000000    10.990000000
 160     0.838389694     0.000000000    10.990000000
```

TABLE E.8

CONFIGURATION ONE: PRESSURE AND VELOCITY RESULTS FOR VISCOSITY=10,000 CP AND N=5

```
INITIAL VISCOSITY OF THE OIL= 10000.000 CP
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT= 25


COLLOCATION   PRESSURE        VELOCITY IN     VELOCITY IN
   POINT        (PSI)         X DIRECTION     Y DIRECTION
                                 CM/S            CM/S

     1      124.458881045     0.000000000    10.990000000
     2       74.741527306    -0.000000000    10.990000000
     3       38.631989630    -0.000000000    10.990000000
     4       22.627962278    -0.000000000    10.990000000
     5       14.683383180     0.000000000    10.990000000
     6       12.098182456    -0.000000000    10.990000000
     7        7.732727116    -0.000000000    10.990000000
     8        3.640092418     0.000000000    10.990000000
     9        1.538850295    -0.000000000    10.990000000
    10        1.022078701     0.000000000    10.990000000
    11      124.414753663    10.990000000     0.000000000
    12      120.097658914     0.000008310     0.000008218
    13       73.565799546     0.000002306     0.000011613
    14       38.447714599     0.000000389     0.000002910
    15       22.499623668     0.000000262     0.000002984
    16       14.639435781     0.000000097     0.000001210
    17       13.413418714    10.990000000     0.000001501
    18       12.038278684     0.000000124     0.000001373
    19        7.690142419     0.000000089     0.000000966
    20        3.610432935     0.000000062     0.000000544
    21        1.515324419     0.000000049     0.000000231
    22        1.000000000     0.000000046     0.000000046
    23        0.977951365    10.990000000     0.000000000
    24       73.675806608    10.990000000    -0.000000000
    25       72.544116541     0.000012065     0.000002214
    26       57.009128064     0.000005339     0.000004887
    27       34.179353655     0.000001817     0.000003230
    28       19.982904717     0.000001019     0.000002081
    29       13.317150316     0.000000624     0.000001474
    30       11.965817958    10.990000000     0.000001357
    31       10.717865185     0.000000567     0.000001251
    32        6.721603201     0.000000422     0.000000894
    33        2.930995060     0.000000297     0.000000507
    34        0.975512972     0.000000236     0.000000216
    35        0.493642476     0.000000222     0.000000043
    36        0.473130705    10.990000000     0.000000000
    37       33.620891512    10.990000000    -0.000000000
    38       33.481674198     0.000003895     0.000000294
```

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39       30.253820110        0.000004206        0.000001358
40       20.997036414        0.000002731        0.000001769
41       12.428200200        0.000001663        0.000001317
42        8.004164070        0.000001244        0.000001047
43        7.039032061       10.990000000        0.000000965
44        6.149146754        0.000001062        0.000000895
45        3.259720423        0.000000815        0.000000653
46        0.464666887        0.000000586        0.000000377
47       -0.994534404        0.000000469        0.000000162
48       -1.355602302        0.000000441        0.000000032
49       -1.371000490       10.990000000        0.0
50       10.680283701       10.990000000        0.000000000
51       10.602162867        0.000004532        0.000000157
52        9.233437204        0.000003593        0.000000517
53        5.862814807        0.000002736        0.000000663
54        2.330179235        0.000001960        0.000000595
55        0.310472677        0.000001515        0.000000479
56       -0.134511124       10.990000000        0.000000451
57       -0.553147374        0.000001339        0.000000423
58       -1.938027743        0.000001048        0.000000317
59       -3.305658143        0.000000766        0.000000186
60       -4.027853133        0.000000618        0.000000080
61       -4.207306010        0.000000581        0.000000016
62       -4.214939036       10.990000000        0.0
63       -3.892618061       10.990000000        0.000000000
64       -3.875066439        0.000003202       -0.000000031
65       -3.839793763        0.000003382        0.000000041
66       -4.434752893        0.000002756        0.000000143
67       -5.144541571        0.000002009        0.000000111
68       -5.545687421        0.000001591        0.000000102
69       -5.639495883       10.990000000        0.000000092
70       -5.725052631        0.000001403        0.000000086
71       -6.008155950        0.000001108        0.000000065
72       -6.289757274        0.000000814        0.000000038
73       -6.439061054        0.000000658        0.000000017
74       -6.476204286        0.000000619        0.000000003
75       -6.477799470       10.990000000       -0.000000000
76       -7.070464576        0.000003623       10.990000000
77       -7.070463162        0.000003374       10.990000000
78       -7.070461145        0.000002745       10.990000000
79       -7.070458416        0.000002013       10.990000000
80       -7.070456736        0.000001591       10.990000000
81       -7.070455249        0.000001407       10.990000000
82       -7.070451205        0.000001110       10.990000000
83       -7.070446477        0.000000816       10.990000000
84       -7.070443373        0.000000660       10.990000000
85       -7.070443284        0.000000621       10.990000000
86      -10.248311126       10.990000000       -0.000000000
87      -10.265862037        0.000003202        0.000000031
88      -10.301131885        0.000003382       -0.000000041
89       -9.706168719        0.000002756       -0.000000143
90       -8.996374578        0.000002009       -0.000000111
91       -8.595225362        0.000001591       -0.000000102
92       -8.501416100       10.990000000       -0.000000092
93       -8.415857167        0.000001403       -0.000000086
94       -8.132745751        0.000001108       -0.000000065
95       -7.851134925        0.000000814       -0.000000038
96       -7.701824911        0.000000658       -0.000000017
97       -7.664681762        0.000000619       -0.000000003
98       -7.663087180       10.990000000        0.000000000
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CONTINUED

  99     -24.821210896      10.990000000      -0.000000000
 100     -24.743089351       0.000004532      -0.000000157
 101     -23.374360849       0.000003593      -0.000000517
 102     -20.003734370       0.000002736      -0.000000663
 103     -16.471093216       0.000001960      -0.000000595
 104     -14.451383127       0.000001515      -0.000000479
 105     -14.006398469      10.990000000      -0.000000451
 106     -13.587759972       0.000001339      -0.000000423
 107     -12.202871147       0.000001048      -0.000000317
 108     -10.835230449       0.000000766      -0.000000186
 109     -10.113028107       0.000000618      -0.000000080
 110      -9.933574453       0.000000581      -0.000000016
 111      -9.925941961      10.990000000       0.000000000
 112     -47.761818029      10.990000000      -0.000000000
 113     -47.622600004       0.000003895      -0.000000294
 114     -44.394743041       0.000004206      -0.000001358
 115     -35.137955102       0.000002731      -0.000001769
 116     -26.569112900       0.000001663      -0.000001317
 117     -22.145072683       0.000001244      -0.000001047
 118     -21.179939616      10.990000000      -0.000000965
 119     -20.290051823       0.000001062      -0.000000895
 120     -17.400615807       0.000000815      -0.000000653
 121     -14.605548662       0.000000586      -0.000000377
 122     -13.146335400       0.000000469      -0.000000162
 123     -12.785265251       0.000000441      -0.000000032
 124     -12.769867553      10.990000000       0.0
 125     -87.816733496      10.990000000       0.000000000
 126     -86.685042715       0.000012065      -0.000002214
 127     -71.150051319       0.000005339      -0.000004887
 128     -48.320272489       0.000001817      -0.000003230
 129     -34.123817128       0.000001019      -0.000002081
 130     -27.458058037       0.000000624      -0.000001474
 131     -26.106724403      10.990000000      -0.000001357
 132     -24.858768903       0.000000567      -0.000001251
 133     -20.862495563       0.000000422      -0.000000894
 134     -17.071868926       0.000000297      -0.000000507
 135     -15.116361526       0.000000236      -0.000000216
 136     -14.634476264       0.000000222      -0.000000043
 137     -14.613963967      10.990000000      -0.000000000
 138    -138.555682448      10.990000000      -0.000000000
 139    -134.238586982       0.000008310      -0.000008218
 140     -87.706724664       0.000002306      -0.000011613
 141     -52.588635223       0.000000389      -0.000002910
 142     -36.640537721       0.000000262      -0.000002984
 143     -28.780344915       0.000000097      -0.000001210
 144     -27.554326479      10.990000000      -0.000001501
 145     -26.179183890       0.000000124      -0.000001373
 146     -21.831035193       0.000000089      -0.000000966
 147     -17.751305627       0.000000062      -0.000000544
 148     -15.656159264       0.000000049      -0.000000231
 149     -15.140788099       0.000000046      -0.000000046
 150     -15.118735649      10.990000000      -0.000000000
 151    -138.599809785       0.000000000      10.990000000
 152     -88.882453093       0.000000000      10.990000000
 153     -52.772910920      -0.000000000      10.990000000
 154     -36.768876989       0.0              10.990000000
 155     -28.824292960       0.000000000      10.990000000
 156     -26.239088328      -0.000000000      10.990000000
 157     -21.873620476       0.0              10.990000000
 158     -17.780965647       0.000000000      10.990000000
 159     -15.679684660      -0.000000000      10.990000000
 160     -15.162863030       0.0              10.990000000
```

TABLE E.9

CONFIGURATION TWO: PRESSURE AND VELOCITY RESULTS FOR VISCOSITY=100 CP AND N=2

INITIAL VISCOSITY OF THE OIL= 100.000 CP
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT= 4

COLLOCATION POINT	PRESSURE (PSI)	VELOCITY IN X DIRECTION CM/S	VELOCITY IN Y DIRECTION CM/S
1	1.699253216	-0.000000000	10.990000000
2	1.320997053	0.000000000	10.990000000
3	1.124978045	-0.000000000	10.990000000
4	1.008670703	-0.000000000	10.990000000
5	1.699253216	10.990000000	-0.000000000
6	1.655068647	0.000001915	0.000001915
7	1.302932906	0.000000796	0.000003065
8	1.191017014	10.990000000	0.000001991
9	1.114614987	0.000000469	0.000001553
10	1.000000000	0.000000404	0.000000404
11	0.991329297	10.990000000	0.000000000
12	1.320997053	10.990000000	-0.000000000
13	1.302932906	0.000003065	0.000000796
14	1.136812167	0.000001716	0.000001716
15	1.064555539	10.990000000	0.000001593
16	1.000000000	0.000001388	0.000001388
17	0.885385013	0.000001553	0.000000469
18	0.875021955	10.990000000	-0.000000000
19	1.191017014	0.000001991	10.990000000
20	1.064555539	0.000001593	10.990000000
21	0.935444461	0.000001593	10.990000000
22	0.808982986	0.000001991	10.990000000
23	1.124978045	10.990000000	-0.000000000
24	1.114614987	0.000001553	0.000000469
25	1.000000000	0.000001388	0.000001388
26	0.935444461	10.990000000	0.000001593
27	0.863187833	0.000001716	0.000001716
28	0.697067094	0.000003065	0.000000796
29	0.679002947	10.990000000	0.0
30	1.008670703	10.990000000	0.000000000
31	1.000000000	0.000000404	0.000000404
32	0.885385013	0.000000469	0.000001553
33	0.808982986	10.990000000	0.000001991
34	0.697067094	0.000000796	0.000003065
35	0.344931353	0.000001915	0.000001915
36	0.300746784	10.990000000	0.0
37	0.991329297	-0.000000000	10.990000000
38	0.875021955	-0.000000000	10.990000000
39	0.679002947	0.0	10.990000000
40	0.300746784	0.0	10.990000000

TABLE E.10

CONFIGURATION TWO: PRESSURE AND VELOCITY RESULTS FOR VISCOSITY=10,000 CP AND N=2

```
INITIAL VISCOSITY OF THE OIL= 10000.000 CP
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT=  4


COLLOCATION   PRESSURE        VELOCITY IN      VELOCITY IN
   POINT       (PSI)          X DIRECTION      Y DIRECTION
                                 CM/S             CM/S

     1       70.925321552    -0.000000000     10.990000000
     2       33.099705303     0.000000000     10.990000000
     3       13.497804492    -0.000000000     10.990000000
     4        1.867070269     0.0             10.990000000
     5       70.925321552    10.990000000     -0.000000000
     6       66.506864720     0.000001915      0.000001915
     7       31.293290627     0.000000796      0.000003065
     8       20.101701372    10.990000000      0.000001991
     9       12.461498682     0.000000469      0.000001553
    10        1.000000000     0.000000404      0.000000404
    11        0.132929731    10.990000000     -0.000000000
    12       33.099705303    10.990000000     -0.000000000
    13       31.293290627     0.000003065      0.000000796
    14       14.681216660     0.000001716      0.000001716
    15        7.455553897    10.990000000      0.000001593
    16        1.000000000     0.000001388      0.000001388
    17      -10.461498682     0.000001553      0.000000469
    18      -11.497804492    10.990000000      0.0
    19       20.101701372     0.000001991     10.990000000
    20        7.455553897     0.000001593     10.990000000
    21       -5.455553897     0.000001593     10.990000000
    22      -18.101701372     0.000001991     10.990000000
    23       13.497804492    10.990000000     -0.000000000
    24       12.461498682     0.000001553      0.000000469
    25        1.000000000     0.000001388      0.000001388
    26       -5.455553897    10.990000000      0.000001593
    27      -12.681216660     0.000001716      0.000001716
    28      -29.293290627     0.000003065      0.000000796
    29      -31.099705303    10.990000000      0.000000000
    30        1.867070269    10.990000000      0.0
    31        1.000000000     0.000000404      0.000000404
    32      -10.461498682     0.000000469      0.000001553
    33      -18.101701372    10.990000000      0.000001991
    34      -29.293290627     0.000000796      0.000003065
    35      -64.506864720     0.000001915      0.000001915
    36      -68.925321552    10.990000000      0.0
    37        0.132929731     0.000000000     10.990000000
    38      -11.497804492     0.0             10.990000000
    39      -31.099705303    -0.000000000     10.990000000
    40      -68.925321552     0.0             10.990000000
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TABLE E.11

CONFIGURATION TWO: PRESSURE AND VELOCITY RESULTS FOR VISCOSITY=100 CP AND N=3

```
INITIAL VISCOSITY OF THE OIL=   100.000 CP
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT=  9
```

COLLOCATION POINT	PRESSURE (PSI)	VELOCITY IN X DIRECTION CM/S	VELOCITY IN Y DIRECTION CM/S
1	1.988869925	-0.000000000	10.990000000
2	1.533457832	0.000000000	10.990000000
3	1.251182955	0.000000000	10.990000000
4	1.160393302	0.000000000	10.990000000
5	1.050134430	0.000000000	10.990000000
6	1.002598610	-0.000000000	10.990000000
7	1.988869953	10.990000000	-0.000000000
8	1.945329422	0.000003486	0.000003486
9	1.520076363	0.000001097	0.000005256
10	1.247282129	0.000000334	0.000001980
11	1.202126004	10.990000000	0.000002102
12	1.157051901	0.000000286	0.000001820
13	1.047500608	0.000000228	0.000000979
14	1.000000000	0.000000225	0.000000225
15	0.997400451	10.990000000	0.000000000
16	1.533458380	10.990000000	0.000000000
17	1.520076882	0.000005256	0.000001097
18	1.359682893	0.000002443	0.000002443
19	1.183024121	0.000001215	0.000001920
20	1.140728013	10.990000000	0.000001771
21	1.101913591	0.000001052	0.000001602
22	1.000006148	0.000000960	0.000000960
23	0.952508463	0.000000979	0.000000228
24	0.949874364	10.990000000	0.000000000
25	1.251184709	10.990000000	0.000000000
26	1.247283846	0.000001980	0.000000334
27	1.183025236	0.000001920	0.000001215
28	1.069694732	0.000001547	0.000001547
29	1.034352484	10.990000000	0.000001515
30	1.000008603	0.000001465	0.000001465
31	0.898101992	0.000001602	0.000001052
32	0.842962680	0.000001820	0.000000286
33	0.839621215	10.990000000	0.000000000
34	1.202128214	0.000002102	10.990000000
35	1.140729552	0.000001771	10.990000000
36	1.034352859	0.000001515	10.990000000
37	0.965665180	0.000001515	10.990000000
38	0.859288295	0.000001771	10.990000000

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39     0.797889409    0.000002102   10.990000000
40     1.160396049   10.990000000   -0.000000000
41     1.157054605    0.000001820    0.000000286
42     1.101915593    0.000001602    0.000001052
43     1.000009365    0.000001465    0.000001465
44     0.965665556   10.990000000    0.000001515
45     0.930323350    0.000001547    0.000001547
46     0.816992827    0.000001920    0.000001215
47     0.752734110    0.000001980    0.000000334
48     0.748833237   10.990000000    0.0
49     1.050140264   10.990000000    0.000000000
50     1.047506295    0.000000979    0.000000228
51     1.000010081    0.000000960    0.000000960
52     0.898103994    0.000001052    0.000001602
53     0.859289834   10.990000000    0.000001771
54     0.816993942    0.000001215    0.000001920
55     0.640335622    0.000002443    0.000002443
56     0.479941778    0.000005256    0.000001097
57     0.466560287   10.990000000    0.0
58     1.002609424   10.990000000    0.000000000
59     1.000010338    0.000000225    0.000000225
60     0.952514151    0.000000228    0.000000979
61     0.842965384    0.000000286    0.000001820
62     0.797891619   10.990000000    0.000002102
63     0.752735827    0.000000334    0.000001980
64     0.479942297    0.000001097    0.000005256
65     0.054689489    0.000003486    0.000003486
66     0.011148970   10.990000000   -0.000000000
67     0.997411265   -0.000000000   10.990000000
68     0.949880198    0.000000000   10.990000000
69     0.839623961   -0.000000000   10.990000000
70     0.748834991   -0.000000000   10.990000000
71     0.466560835    0.000000000   10.990000000
72     0.011148999   -0.000000000   10.990000000
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TABLE E.12

CONFIGURATION TWO: PRESSURE AND VELOCITY RESULTS FOR VISCOSITY=10,000 CP AND N=3

INITIAL VISCOSITY OF THE OIL= 10000.000 CP
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT= 9

COLLOCATION POINT	PRESSURE (PSI)	VELOCITY IN X DIRECTION CM/S	VELOCITY IN Y DIRECTION CM/S
1	99.886057167	-0.000000000	10.990000000
2	54.344886312	0.000000000	10.990000000
3	26.117493975	-0.000000000	10.990000000
4	17.038611645	-0.000000000	10.990000000
5	6.013007688	0.000000000	10.990000000
6	1.259907488	0.0	10.990000000
7	99.886057195	10.990000000	-0.000000000
8	95.532006103	0.000003486	0.000003486
9	53.006738325	0.000001097	0.000005256
10	25.727409062	0.000000334	0.000001980
11	21.211837451	10.990000000	0.000002102
12	16.704468313	0.000000286	0.000001820
13	5.749611750	0.000000228	0.000000979
14	1.000000000	0.000000225	0.000000225
15	0.740091572	10.990000000	0.000000000
16	54.344886860	10.990000000	-0.000000000
17	53.006738844	0.000005256	0.000001097
18	36.967372803	0.000002443	0.000002443
19	19.301573170	0.000001215	0.000001920
20	15.071994059	10.990000000	0.000001771
21	11.190587763	0.000001052	0.000001602
22	1.000006148	0.000000960	0.000000960
23	-3.749602679	0.000000979	0.000000228
24	-4.012998895	10.990000000	0.0
25	26.117495729	10.990000000	0.000000000
26	25.727410779	0.000001980	0.000000334
27	19.301574284	0.000001920	0.000001215
28	7.968578179	0.000001547	0.000001547
29	4.434374003	10.990000000	0.000001515
30	1.000008603	0.000001465	0.000001465
31	-9.190572179	0.000001602	0.000001052
32	-14.704453732	0.000001820	0.000000286
33	-15.038597128	10.990000000	-0.000000000
34	21.211839662	0.000002102	10.990000000
35	15.071995598	0.000001771	10.990000000
36	4.434374379	0.000001515	10.990000000

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37      -2.434356339       0.000001515      10.990000000
38     -13.071977751       0.000001771      10.990000000
39     -19.211822039       0.000002102      10.990000000
40      17.038614391      10.990000000       0.000000000
41      16.704471017       0.000001820       0.000000286
42      11.190589764       0.000001602       0.000001052
43       1.000009365       0.000001465       0.000001465
44      -2.434355964      10.990000000       0.000001515
45      -5.968560097       0.000001547       0.000001547
46     -17.301556221       0.000001920       0.000001215
47     -23.727392823       0.000001980       0.000000334
48     -24.117477783      10.990000000      -0.000000000
49       6.013013522      10.990000000      -0.000000000
50       5.749617437       0.000000979       0.000000228
51       1.000010081       0.000000960       0.000000960
52      -9.190570178       0.000001052       0.000001602
53     -13.071976212      10.990000000       0.000001771
54     -17.301555107       0.000001215       0.000001920
55     -34.967354289       0.000002443       0.000002443
56     -51.006720184       0.000005256       0.000001097
57     -52.344868194      10.990000000      -0.000000000
58       1.259918303      10.990000000       0.000000000
59       1.000010338       0.000000225       0.000000225
60      -3.749596992       0.000000228       0.000000979
61     -14.704451028       0.000000286       0.000001820
62     -19.211819829      10.990000000       0.000002102
63     -23.727391106       0.000000334       0.000001980
64     -51.006719665       0.000001097       0.000005256
65     -93.531987192       0.000003486       0.000003486
66     -97.886038271      10.990000000       0.0
67       0.740102386      -0.000000000      10.990000000
68      -4.012993061       0.000000000      10.990000000
69     -15.038594381      -0.000000000      10.990000000
70     -24.117476029       0.0              10.990000000
71     -52.344867646       0.000000000      10.990000000
72     -97.886038243       0.0              10.990000000
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TABLE E.13

CONFIGURATION TWO: PRESSURE AND VELOCITY RESULTS FOR VISCOSITY=100 CP AND N=4

INITIAL VISCOSITY OF THE OIL= 100.000 CP
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT= 16

COLLOCATION POINT	PRESSURE (PSI)	VELOCITY IN X DIRECTION CM/S	VELOCITY IN Y DIRECTION CM/S
1	2.213532493	0.000000000	10.990000000
2	1.726977279	-0.000000000	10.990000000
3	1.378588361	-0.000000000	10.990000000
4	1.240539404	-0.000000000	10.990000000
5	1.176301182	-0.000000000	10.990000000
6	1.090222678	0.000000000	10.990000000
7	1.021792722	0.000000000	10.990000000
8	1.000961063	0.000000000	10.990000000
9	2.213532493	10.990000000	0.000000000
10	2.170096360	0.000005608	0.000005608
11	1.714862171	0.000001606	0.000008176
12	1.376363900	0.000000314	0.000002555
13	1.238655397	0.000000259	0.000002724
14	1.203495726	10.990000000	0.000002094
15	1.175085393	0.000000172	0.000001917
16	1.089189332	0.000000146	0.000001329
17	1.020827231	0.000000136	0.000000648
18	1.000000000	0.000000136	0.000000136
19	0.999038937	10.990000000	0.000000000
20	1.726977279	10.990000000	-0.000000000
21	1.714862171	0.000008176	0.000001606
22	1.554513969	0.000003613	0.000003613
23	1.330837258	0.000001332	0.000002684
24	1.204798417	0.000000949	0.000002108
25	1.175979201	10.990000000	0.000001950
26	1.149383240	0.000000777	0.000001804
27	1.067259357	0.000000671	0.000001292
28	1.000000000	0.000000645	0.000000645
29	0.979172769	0.000000648	0.000000136
30	0.978207278	10.990000000	0.0
31	1.378588361	10.990000000	-0.000000000
32	1.376363900	0.000002555	0.000000314
33	1.330837258	0.000002684	0.000001332
34	1.215042441	0.000001830	0.000001830
35	1.120894419	0.000001403	0.000001674
36	1.097525165	10.990000000	0.000001627
37	1.074937508	0.000001308	0.000001559
38	1.000000000	0.000001241	0.000001241

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39    0.932740643    0.000001292    0.000000671
40    0.910810668    0.000001329    0.000000146
41    0.909777322   10.990000000    0.0
42    1.240539404   10.990000000   -0.000000000
43    1.238659397    0.000002724    0.000000259
44    1.204798417    0.000002108    0.000000949
45    1.120894419    0.000001674    0.000001403
46    1.042499703    0.000001510    0.000001510
47    1.021137229   10.990000000    0.000001501
48    1.000000000    0.000001480    0.000001480
49    0.925062492    0.000001559    0.000001308
50    0.850616760    0.000001804    0.000000777
51    0.824914607    0.000001917    0.000000172
52    0.823698818   10.990000000    0.0
53    1.203495726    0.000002094   10.990000000
54    1.175979201    0.000001950   10.990000000
55    1.097525165    0.000001627   10.990000000
56    1.021137229    0.000001501   10.990000000
57    0.978862771    0.000001501   10.990000000
58    0.902474835    0.000001627   10.990000000
59    0.824020799    0.000001950   10.990000000
60    0.796504274    0.000002094   10.990000000
61    1.176301182   10.990000000   -0.000000000
62    1.175085393    0.000001917    0.000000172
63    1.149383240    0.000001804    0.000000777
64    1.074937508    0.000001559    0.000001308
65    1.000000000    0.000001480    0.000001480
66    0.978862771   10.990000000    0.000001501
67    0.957500297    0.000001510    0.000001510
68    0.879105581    0.000001674    0.000001403
69    0.795201583    0.000002108    0.000000949
70    0.761340603    0.000002724    0.000000259
71    0.759460596   10.990000000   -0.000000000
72    1.090222678   10.990000000   -0.000000000
73    1.089189332    0.000001329    0.000000146
74    1.067259357    0.000001292    0.000000671
75    1.000000000    0.000001241    0.000001241
76    0.925062492    0.000001308    0.000001559
77    0.902474835   10.990000000    0.000001627
78    0.879105581    0.000001403    0.000001674
79    0.784957559    0.000001830    0.000001830
80    0.669162742    0.000002684    0.000001332
81    0.623636100    0.000002555    0.000000314
82    0.621411639   10.990000000   -0 000000000
83    1.021792722   10.990000000    0.000000000
84    1.020827231    0.000000648    0.000000136
85    1.000000000    0.000000645    0.000000645
86    0.932740643    0.000000671    0.000001292
87    0.850616760    0.000000777    0.000001804
88    0.824020799   10.990000000    0.000001950
89    0.795201583    0.000000949    0.000002108
90    0.669162742    0.000001332    0.000002684
91    0.445486031    0.000003613    0.000003613
92    0.285137829    0.000008176    0.000001606
93    0.273022721   10.990000000    0.0
94    1.000961063   10.990000000   -0.000000000
95    1.000000000    0.000000136    0.000000136
96    0.979172769    0.000000136    0.000000648
97    0.910810668    0.000000146    0.000001329
98    0.824914607    0.000000172    0.000001917
```

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CONTINUED


  99       0.796504274      10.990000000       0.000002094
 100       0.761340603       0.000000259       0.000002724
 101       0.623636100       0.000000314       0.000002555
 102       0.285137829       0.000001606       0.000008176
 103      -0.170096360       0.000005608       0.000005608
 104      -0.213532493      10.990000000      -0.000000000
 105       0.999038937       0.0              10.990000000
 106       0.978207278      -0.000000000      10.990000000
 107       0.909777322       0.000000000      10.990000000
 108       0.823698818      -0.000000000      10.990000000
 109       0.759460596      -0.000000000      10.990000000
 110       0.621411639       0.0              10.990000000
 111       0.273022721       0.0              10.990000000
 112      -0.213532493      -0.000000000      10.990000000
```

TABLE E.14

CONFIGURATION TWO: PRESSURE AND VELOCITY RESULTS FOR VISCOSITY=10,000 CP AND N=4

INITIAL VISCOSITY OF THE OIL= 10000.000 CP
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT= 16

COLLOCATION POINT	PRESSURE (PSI)	VELOCITY IN X DIRECTION CM/S	VELOCITY IN Y DIRECTION CM/S
1	122.353249324	0.000000000	10.990000000
2	73.697727881	-0.000000000	10.990000000
3	38.858836077	-0.000000000	10.990000000
4	25.053940375	0.000000000	10.990000000
5	18.630118188	0.000000000	10.990000000
6	10.022267763	0.000000000	10.990000000
7	3.179272213	-0.000000000	10.990000000
8	1.096106318	0.0	10.990000000
9	122.353249324	10.990000000	0.000000000
10	118.009636043	0.000005608	0.000005608
11	72.486217099	0.000001606	0.000008176
12	38.636390025	0.000000314	0.000002555
13	24.865939747	0.000000259	0.000002724
14	21.349572601	10.990000000	0.000002094
15	18.508539280	0.000000172	0.000001917
16	9.918933218	0.000000146	0.000001329
17	3.082723051	0.000000136	0.000000648
18	1.000000000	0.000000136	0.000000136
19	0.903893682	10.990000000	0.000000000
20	73.697727881	10.990000000	-0.000000000
21	72.486217099	0.000008176	0.000001606
22	56.451396891	0.000003613	0.000003613
23	34.083725767	0.000001332	0.000002684
24	21.479841662	0.000000949	0.000002108
25	18.597920077	10.990000000	0.000001950
26	15.938324045	0.000000777	0.000001804
27	7.725935690	0.000000671	0.000001292
28	1.000000000	0.000000645	0.000000645
29	-1.082723051	0.000000648	0.000000136
30	-1.179272213	10.990000000	-0.000000000
31	38.858836077	10.990000000	-0.000000000
32	38.636390025	0.000002555	0.000000314
33	34.083725767	0.000002684	0.000001332
34	22.504244097	0.000001830	0.000001830
35	13.089441872	0.000001403	0.000001674
36	10.752516483	10.990000000	0.000001627
37	8.493750800	0.000001308	0.000001559
38	1.000000000	0.000001241	0.000001241
39	-5.725935690	0.000001292	0.000000671

CONTINUED

```
40      -7.918933218     0.000001329     0.000000146
41      -8.022267763    10.990000000     0.000000000
42      25.053940375    10.990000000     0.000000000
43      24.865939747     0.000002724     0.000000259
44      21.479841662     0.000002108     0.000000949
45      13.089441872     0.000001674     0.000001403
46       5.249970285     0.000001510     0.000001510
47       3.113722881    10.990000000     0.000001501
48       1.000000000     0.000001480     0.000001480
49      -6.493750800     0.000001559     0.000001308
50     -13.938324045     0.000001804     0.000000777
51     -16.508539280     0.000001917     0.000000172
52     -16.630118188    10.990000000     0.0
53      21.349572601     0.000002094    10.990000000
54      18.597920077     0.000001950    10.990000000
55      10.752516483     0.000001627    10.990000000
56       3.113722881     0.000001501    10.990000000
57      -1.113722881     0.000001501    10.990000000
58      -8.752516483     0.000001627    10.990000000
59     -16.597920077     0.000001950    10.990000000
60     -19.349572601     0.000002094    10.990000000
61      18.630118188    10.990000000    -0.000000000
62      18.508539280     0.000001917     0.000000172
63      15.938324045     0.000001804     0.000000777
64       8.493750800     0.000001559     0.000001308
65       1.000000000     0.000001480     0.000001480
66      -1.113722881    10.990000000     0.000001501
67      -3.249970285     0.000001510     0.000001510
68     -11.089441872     0.000001674     0.000001403
69     -19.479841662     0.000002108     0.000000949
70     -22.865939747     0.000002724     0.000000259
71     -23.053940375    10.990000000    -0.000000000
72      10.022267763    10.990000000     0.000000000
73       9.918933218     0.000001329     0.000000146
74       7.725935690     0.000001292     0.000000671
75       1.000000000     0.000001241     0.000001241
76      -6.493750800     0.000001308     0.000001559
77      -8.752516483    10.990000000     0.000001627
78     -11.089441872     0.000001403     0.000001674
79     -20.504244097     0.000001830     0.000001830
80     -32.083725767     0.000002684     0.000001332
81     -36.636390025     0.000002555     0.000000314
82     -36.858836077    10.990000000     0.000000000
83       3.179272213    10.990000000    -0.000000000
84       3.082723051     0.000000648     0.000000136
85       1.000000000     0.000000645     0.000000645
86      -5.725935690     0.000000671     0.000001292
87     -13.938324045     0.000000777     0.000001804
88     -16.597920077    10.990000000     0.000001950
89     -19.479841662     0.000000949     0.000002108
90     -32.083725767     0.000001332     0.000002684
91     -54.451396891     0.000003613     0.000003613
92     -70.486217099     0.000008176     0.000001606
93     -71.697727881    10.990000000     0.0
94       1.096106318    10.990000000    -0.000000000
95       1.000000000     0.000000136     0.000000136
96      -1.082723051     0.000000136     0.000000648
97      -7.918933218     0.000000146     0.000001329
98     -16.508539280     0.000000172     0.000001917
99     -19.349572601    10.990000000     0.000002094
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CONTINUED


100     -22.865939747      0.000000259       0.000002724
101     -36.636390025      0.000000314       0.000002555
102     -70.486217099      0.000001606       0.000008176
103    -116.009636043      0.000005608       0.000005608
104    -120.353249324     10.990000000       0.000000000
105       0.903893682      0.000000000      10.990000000
106      -1.179272213     -0.000000000      10.990000000
107      -8.022267763      0.0              10.990000000
108     -16.630118188      0.0              10.990000000
109     -23.053940375      0.0              10.990000000
110     -36.858836077      0.000000000      10.990000000
111     -71.697727881      0.0              10.990000000
112    -120.353249324      0.0              10.990000000
```

TABLE E.15

CONFIGURATION TWO: PRESSURE AND VELOCITY RESULTS FOR VISCOSITY=100 CP AND N=5

INITIAL VISCOSITY OF THE OIL= 100.000 CP
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT= 25

COLLOCATION POINT	PRESSURE (PSI)	VELOCITY IN X DIRECTION CM/S	VELOCITY IN Y DIRECTION CM/S
1	2.395794653	0.000000000	10.990000000
2	1.893573156	-0.000000000	10.990000000
3	1.514036351	0.000000000	10.990000000
4	1.325556066	-0.000000000	10.990000000
5	1.223481067	0.000000000	10.990000000
6	1.185775722	-0.000000000	10.990000000
7	1.119491111	0.000000000	10.990000000
8	1.050122807	0.000000000	10.990000000
9	1.010663730	-0.000000000	10.990000000
10	1.000440693	-0.000000000	10.990000000
11	2.395794659	10.990000000	0.000000000
12	2.352402911	0.000008264	0.000008264
13	1.882020956	0.000002263	0.000011834
14	1.512347532	0.000000356	0.000003351
15	1.324348961	0.000000246	0.000003566
16	1.223057498	0.000000094	0.000001829
17	1.204854747	10.990000000	0.000002122
18	1.185160689	0.000000128	0.000001993
19	1.118988901	0.000000105	0.000001547
20	1.049672205	0.000000094	0.000000985
21	1.010223472	0.000000092	0.000000453
22	1.000000000	0.000000092	0.000000092
23	0.999558146	10.990000000	0.000000000
24	1.893573310	10.990000000	-0.000000000
25	1.882021104	0.000011834	0.000002263
26	1.721273002	0.000005123	0.000005123
27	1.473274547	0.000001655	0.000003699
28	1.300976262	0.000000938	0.000002699
29	1.210206070	0.000000608	0.000002132
30	1.190378752	10.990000000	0.000002017
31	1.171585111	0.000000583	0.000001909
32	1.107509133	0.000000502	0.000001512
33	1.039267538	0.000000459	0.000000976
34	1.000008814	0.000000452	0.000000452
35	0.989789833	0.000000452	0.000000092
36	0.989349338	10.990000000	0.0
37	1.514037077	10.990000000	-0.000000000
38	1.512348252	0.000003351	0.000000356

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CONTINUED

39        1.473275116     0.000003699     0.000001655
40        1.356043675     0.000002354     0.000002354
41        1.232651510     0.000001477     0.000002083
42        1.158569700     0.000001205     0.000001861
43        1.141111357    10.990000000     0.000001781
44        1.124405368     0.000001101     0.000001709
45        1.065669113     0.000001001     0.000001419
46        1.000013753     0.000000962     0.000000962
47        0.960756949     0.000000976     0.000000459
48        0.950351500     0.000000985     0.000000094
49        0.949900948    10.990000000     0.000000000
50        1.325557798    10.990000000     0.000000000
51        1.324350686     0.000003566     0.000000246
52        1.300977821     0.000002699     0.000000938
53        1.232652462     0.000002083     0.000001477
54        1.145348172     0.000001643     0.000001643
55        1.084449507     0.000001450     0.000001586
56        1.069376634    10.990000000     0.000001562
57        1.054567129     0.000001404     0.000001531
58        1.000016304     0.000001365     0.000001365
59        0.934361541     0.000001419     0.000001001
60        0.892520005     0.000001512     0.000000502
61        0.881039932     0.000001547     0.000000105
62        0.880537787    10.990000000    -0.000000000
63        1.223483761    10.990000000    -0.000000000
64        1.223060182     0.000001829     0.000000094
65        1.210208558     0.000002132     0.000000608
66        1.158571503     0.000001861     0.000001205
67        1.084450298     0.000001586     0.000001450
68        1.028719557     0.000001505     0.000001505
69        1.014327392    10.990000000     0.000001499
70        1.000017639     0.000001489     0.000001489
71        0.945466957     0.000001531     0.000001404
72        0.875627286     0.000001709     0.000001101
73        0.828446585     0.000001909     0.000000583
74        0.814871028     0.000001993     0.000000128
75        0.814256082    10.990000000     0.000000000
76        1.204857717     0.000002122    10.990000000
77        1.190381512     0.000002017    10.990000000
78        1.141113403     0.000001781    10.990000000
79        1.069377639     0.000001562    10.990000000
80        1.014327600     0.000001499    10.990000000
81        0.985708205     0.000001499    10.990000000
82        0.930657831     0.000001562    10.990000000
83        0.858921785     0.000001781    10.990000000
84        0.809653523     0.000002017    10.990000000
85        0.795177506     0.000002122    10.990000000
86        1.185778962    10.990000000     0.000000000
87        1.185163923     0.000001993     0.000000128
88        1.171588159     0.000001909     0.000000583
89        1.124407667     0.000001709     0.000001101
90        1.054568353     0.000001531     0.000001404
91        1.000018055     0.000001489     0.000001489
92        0.985708412    10.990000000     0.000001499
93        0.971316162     0.000001505     0.000001505
94        0.915585137     0.00000158[illegible]     0.000001450
95        0.841463719     0.000001861     0.000001205
96        0.789826564     0.000002132     0.000000608
97        0.776975145     0.000001829     0.000000094
98        0.776551658    10.990000000     0.0
```

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CONTINUED

 99     1.119495831     10.990000000     -0.000000000
100     1.118993604      0.000001547      0.000000105
101     1.107513497      0.000001512      0.000000502
102     1.065672487      0.000001419      0.000001001
103     1.000018411      0.000001365      0.000001365
104     0.945468181      0.000001404      0.000001531
105     0.930658837     10.990000000      0.000001562
106     0.915585928      0.000001450      0.000001586
107     0.854687162      0.000001643      0.000001643
108     0.767382895      0.000002083      0.000001477
109     0.699057603      0.000002699      0.000000938
110     0.675684997      0.000003566      0.000000246
111     0.674477979     10.990000000     -0.000000000
112     1.050130260     10.990000000      0.000000000
113     1.049679634      0.000000985      0.000000094
114     1.039274400      0.000000976      0.000000459
115     1.000018915      0.000000962      0.000000962
116     0.934364915      0.000001001      0.000001419
117     0.875629585      0.000001101      0.000001709
118     0.858923831     10.990000000      0.000001781
119     0.841465522      0.000001205      0.000001861
120     0.767383847      0.000001477      0.000002083
121     0.643991971      0.000002354      0.000002354
122     0.526760763      0.000003699      0.000001655
123     0.487687932      0.000003351      0.000000356
124     0.485999204     10.990000000      0.000000000
125     1.010676686     10.990000000      0.000000000
126     1.010236261      0.000000452      0.000000092
127     1.000019272      0.000000452      0.000000452
128     0.960763811      0.000000459      0.000000976
129     0.892524369      0.000000502      0.000001512
130     0.828449633      0.000000583      0.000001909
131     0.809656284     10.990000000      0.000002017
132     0.789829051      0.000000608      0.000002132
133     0.699059162      0.000000938      0.000002699
134     0.526761332      0.000001655      0.000003699
135     0.278763207      0.000005123      0.000005123
136     0.118015436      0.000011834      0.000002263
137     0.106463326     10.990000000      0.0
138     1.000460932     10.990000000      0.000000000
139     1.000019610      0.000000092      0.000000092
140     0.989802622      0.000000092      0.000000452
141     0.950358929      0.000000094      0.000000985
142     0.881044635      0.000000105      0.000001547
143     0.814874262      0.000000128      0.000001993
144     0.795180476     10.990000000      0.000002122
145     0.776977830      0.000000094      0.000001829
146     0.675686722      0.000000246      0.000003566
147     0.487688653      0.000000356      0.000003351
148     0.118015584      0.000002263      0.000011834
149    -0.352366035      0.000008264      0.000008264
150    -0.395757686     10.990000000      0.0
151     0.999578385     -0.000000000     10.990000000
152     0.989362294      0.0             10.990000000
153     0.949908402      0.0             10.990000000
154     0.880542507      0.0             10.990000000
155     0.814259322      0.000000000     10.990000000
156     0.776554352      0.0             10.990000000
157     0.674479711     -0.000000000     10.990000000
158     0.485999931      0.000000000     10.990000000
159     0.106463481      0.0             10.990000000
160    -0.395757679      0.000000000     10.990000000
```

TABLE E.16

CONFIGURATION TWO: PRESSURE AND VELOCITY RESULTS FOR VISCOSITY=10,000 CP AND N=5

INITIAL VISCOSITY OF THE OIL= 10000.000 CP
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT= 25

COLLOCATION POINT	PRESSURE (PSI)	VELOCITY IN X DIRECTION CM/S	VELOCITY IN Y DIRECTION CM/S
1	140.577633529	0.000000000	10.990000000
2	90.355507869	0.0	10.990000000
3	52.401873336	0.000000000	10.990000000
4	33.553919678	0.000000000	10.990000000
5	23.346485189	-0.000000000	10.990000000
6	19.575996330	0.000000000	10.990000000
7	12.947679224	0.000000000	10.990000000
8	6.011103645	-0.000000000	10.990000000
9	2.065725493	0.0	10.990000000
10	1.044126816	-0.000000000	10.990000000
11	140.577633535	10.990000000	0.000000000
12	136.238463802	0.000008264	0.000008264
13	89.200292361	0.000002263	0.000011834
14	52.232995957	0.000000356	0.000003351
15	33.433213516	0.000000246	0.000003566
16	23.304132330	0.000000094	0.000001829
17	21.483876607	10.990000000	0.000002122
18	19.514497329	0.000000128	0.000001993
19	12.897461466	0.000000105	0.000001547
20	5.966045947	0.000000094	0.000000985
21	2.021687948	0.000000092	0.000000452
22	1.000000000	0.000000092	0.000000092
23	0.955872144	10.990000000	0.000000000
24	90.355508009	10.990000000	0.000000000
25	89.200292495	0.000011834	0.000002263
26	73.125506044	0.000005123	0.000005123
27	48.325705078	0.000001655	0.000003699
28	31.095948159	0.000000938	0.000002699
29	22.018989991	0.000000608	0.000002132
30	20.036275932	10.990000000	0.000002017
31	18.156940570	0.000000583	0.000001909
32	11.749469490	0.000000502	0.000001512
33	4.925540507	0.000000459	0.000000976
34	1.000007940	0.000000452	0.000000452
35	-0.021675946	0.000000452	0.000000092
36	-0.065713696	10.990000000	-0.000000000
37	52.401873993	10.990000000	0.000000000
36	52.232996608	0.000003351	0.000000356

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CONTINUED

39       48.325705593        0.000003699       0.000001655
40       36.602601239        0.000002354       0.000002354
41       24.263446211        0.000001477       0.000002083
42       16.855314089        0.000001205       0.000001861
43       15.109493505       10.990000000       0.000001781
44       13.438918766        0.000001101       0.000001709
45        7.565392380        0.000001001       0.000001419
46        1.000012380        0.000000962       0.000000962
47       -2.925518453        0.000000976       0.000000459
48       -3.966024568        0.000000985       0.000000094
49       -4.011082212       10.990000000       0.0
50       33.553921245       10.990000000       0.000000000
51       33.433215076        0.000003566       0.000000246
52       31.095949569        0.000002699       0.000000938
53       24.263447072        0.000002083       0.000001477
54       15.533066344        0.000001643       0.000001643
55        9.443234044        0.000001450       0.000001586
56        7.935955612       10.990000000       0.000001562
57        6.455023925        0.000001404       0.000001531
58        1.000014659        0.000001365       0.000001365
59       -5.565364805        0.000001419       0.000001001
60       -9.749443267        0.000001512       0.000000502
61      -10.897435491        0.000001547       0.000000105
62      -10.947653181       10.990000000      -0.000000000
63       23.346487625       10.990000000      -0.000000000
64       23.304134758        0.000001829       0.000000094
65       22.018992241        0.000002132       0.000000608
66       16.855315720        0.000001861       0.000001205
67        9.443234760        0.000001586       0.000001450
68        3.870185831        0.000001505       0.000001505
69        2.430975388       10.990000000       0.000001499
70        1.000015866        0.000001489       0.000001489
71       -4.454993273        0.000001531       0.000001404
72      -11.438889393        0.000001709       0.000001101
73      -16.156912049        0.000001909       0.000000583
74      -17.514468762        0.000001993       0.000000128
75      -17.575967675       10.990000000       0.0
76       21.483879293        0.000002122      10.990000000
77       20.036278428        0.000002017      10.990000000
78       15.109495355        0.000001781      10.990000000
79        7.935956522        0.000001562      10.990000000
80        2.430975576        0.000001499      10.990000000
81       -0.430943359        0.000001499      10.990000000
82       -5.935924611        0.000001562      10.990000000
83      -13.109463686        0.000001781      10.990000000
84      -18.036246884        0.000002017      10.990000000
85      -19.483847552        0.000002122      10.990000000
86       19.575999260       10.990000000       0.000000000
87       19.514500254        0.000001993       0.000000128
88       18.156943327        0.000001909       0.000000583
89       13.438920845        0.000001709       0.000001101
90        6.455025032        0.000001531       0.000001404
91        1.000016243        0.000001489       0.000001489
92       -0.430943172       10.990000000       0.000001499
93       -1.870153698        0.000001505       0.000001505
94       -7.443202890        0.000001586       0.000001450
95      -14.855284030        0.000001861       0.000001205
96      -20.018960629        0.000002132       0.000000608
97      -21.304102932        0.000001829       0.000000094
98      -21.346455708       10.990000000       0.000000000
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 99      12.947683493      10.990000000       0.000000000
100      12.897465720       0.000001547       0.000000105
101      11.749473437       0.000001512       0.000000502
102       7.565395431       0.000001419       0.000001001
103       1.000016565       0.000001365       0.000001365
104      -4.454992165       0.000001404       0.000001531
105      -5.935923702      10.990000000       0.000001562
106      -7.443202174       0.000001450       0.000001586
107     -13.533034572       0.000001643       0.000001643
108     -22.263415269       0.000002083       0.000001477
109     -29.095917696       0.000002699       0.000000938
110     -31.433182945       0.000003566       0.000000246
111     -31.553889019      10.990000000       0.000000000
112       6.011110386      10.990000000       0.000000000
113       5.966052666       0.000000985       0.000000094
114       4.925546714       0.000000976       0.000000459
115       1.000017049       0.000000962       0.000000962
116      -5.565361754       0.000001001       0.000001419
117     -11.438887313       0.000001101       0.000001709
118     -13.109461835      10.990000000       0.000001781
119     -14.855282398       0.000001205       0.000001861
120     -22.263414408       0.000001477       0.000002083
121     -34.602569172       0.000002354       0.000002354
122     -46.325673316       0.000003699       0.000001655
123     -50.232964036       0.000003351       0.000000356
124     -50.401841325      10.990000000      -0.000000000
125       2.065737212      10.990000000       0.000000000
126       2.021699516       0.000000452       0.000000092
127       1.000017399       0.000000452       0.000000452
128      -2.925512246       0.000000459       0.000000976
129      -9.749439320       0.000000502       0.000001512
130     -16.156909292       0.000000583       0.000001909
131     -18.036244387      10.990000000       0.000002017
132     -20.018958379       0.000000608       0.000002132
133     -29.095916286       0.000000938       0.000002699
134     -46.325672801       0.000001655       0.000003699
135     -71.125473500       0.000005123       0.000005123
136     -87.200259659       0.000011834       0.000002263
137     -88.355475078      10.990000000       0.000000000
138       1.044145121      10.990000000      -0.000000000
139       1.000017736       0.000000092       0.000000092
140      -0.021664379       0.000000092       0.000000452
141      -3.966017849       0.000000094       0.000000985
142     -10.897431237       0.000000105       0.000001547
143     -17.514465837       0.000000128       0.000001993
144     -19.483844865      10.990000000       0.000002122
145     -21.304100504       0.000000094       0.000001829
146     -31.433181384       0.000000246       0.000003566
147     -50.232963385       0.000000356       0.000003351
148     -87.200259525       0.000002263       0.000011834
149    -134.238430710       0.000008264       0.000008264
150    -138.577600352      10.990000000       0.0
151       0.955890449      -0.000000000      10.990000000
152      -0.065701977      -0.000000000      10.990000000
153      -4.011075471      -0.000000000      10.990000000
154     -10.947648911       0.0              10.990000000
155     -17.575964744       0.0              10.990000000
156     -21.346453271       0.0              10.990000000
157     -31.553887452      -0.000000000      10.990000000
158     -50.401840668       0.0              10.990000000
159     -88.355474939       0.000000000      10.990000000
160    -138.577600346       0.000000000      10.990000000
```

APPENDIX F

Concentration Results from Convection Diffusion Equation

A value of 10.99 in the velocity columns indicates that the velocity at that collocation point was not evaluated.

TABLE F.1

CONFIGURATION ONE: CONCENTRATION RESULTS FOR DISPERSION COEFFICIENT=.0001075 SQ.CM/S AND N=3

7

INITIAL VISCOSITY OF THE OIL= 100.000 CP
KD= 0.0001075SQ.CM/S
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT= 9
TIME=50,000 S
DT=50,000 S

COLLOCATION POINT	PRESSURE (PSI)	VELOCITY IN X DIRECTION CM/S	VELOCITY IN Y DIRECTION CM/S	CONCENTRATION		COLLOCATION POINT
				IMPLICIT METHOD	R-K METHOD	
1	1.828773953	-0.000000000	10.990000000	0.0087388	0.0087286	1
2	1.395770126	0.000000000	10.990000000	0.0000115	0.0000118	2
3	1.156293268	-0.000000000	10.990000000	-0.0000015	-0.0000016	3
4	1.093897308	-0.000000000	10.990000000	0.0000003	-0.0	4
5	1.026436607	0.000000000	10.990000000	-0.0000000	-0.0	5
6	1.001309415	-0.000000000	10.990000000	0.0000000	-0.0	6
7	1.826174900	10.990000000	-0.000000000	0.0087394	0.0087291	7
8	1.783944216	0.000003598	0.000003373	0.0076994	0.0076904	8
9	1.381502060	0.000001174	0.000004810	0.0000101	0.0000104	9
10	1.152174607	0.000000353	0.000001360	-0.0000014	-0.0000014	10
11	1.121433160	10.990000000	0.000001474	0.0002191	0.0002186	11
12	1.090773736	0.000000267	0.000001199	0.0000003	0.0	12
13	1.024689370	0.000000151	0.000000533	-0.0000000	0.0	13
14	1.000000000	0.000000113	0.000000112	0.0000000	0.0	14
15	0.998710335	10.990000000	0.000000000	-0.0000168	-0.0000168	15
16	1.345640597	10.990000000	0.000000000	0.0000137	0.0000141	16
17	1.334006465	0.000005789	0.000000946	0.0000121	0.0000124	17
18	1.203668649	0.000002820	0.000001860	0.0000000	0.0000000	18
19	1.083631182	0.000001307	0.000001115	-0.0000000	-0.0000000	19
20	1.060035134	10.990000000	0.000000955	0.0000003	0.0000004	20
21	1.039920776	0.000000960	0.000000797	0.0000000	0.0	21
22	0.994634706	0.000000563	0.000000377	-0.0000000	0.0	22
23	0.977193298	0.000000445	0.000000077	0.0000000	0.0	23
24	0.976306574	10.990000000	-0.000000000	-0.0000000	-0.0000000	24
25	0.995908837	10.990000000	0.000000000	-0.0000033	-0.0000033	25
26	0.995131576	0.000003179	0.000000067	-0.0000029	-0.0000029	26
27	0.981726407	0.000002717	0.000000250	-0.0000000	-0.0000000	27
28	0.959401095	0.000001736	0.000000271	0.0000000	0.0000000	28
29	0.953659550	10.990000000	0.000000224	-0.0000001	-0.0000001	29
30	0.948916384	0.000001276	0.000000189	-0.0000000	0.0	30
31	0.938014976	0.000000805	0.000000092	0.0000000	0.0	31
32	0.933729126	0.000000621	0.000000019	-0.0000000	0.0	32
33	0.933511264	10.990000000	-0.000000000	0.0000000	0.0000000	33
34	0.919316895	0.000003576	10.990000000	0.0002183	0.0002176	34

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35    0.919316689   0.000002726   10.990000000    0.0000003   0.0000003   35
36    0.919316243   0.000001739   10.990000000   -0.0000000  -0.0000000   36
37    0.919315898   0.000001291   10.990000000    0.0000000  -0.0         37
38    0.919315274   0.000000816   10.990000000   -0.0000000  -0.0         38
39    0.919314863   0.000000629   10.990000000    0.0000000  -0.0         39
40    0.842724974  10.990000000   -0.000000000    0.0000008  -0.0         40
41    0.843502213   0.000003179   -0.000000067    0.0000007   0.0         41
42    0.856906969   0.000002717   -0.000000254    0.0000000   0.0         42
43    0.879231393   0.000001736   -0.000000271   -0.0000000   0.0         43
44    0.884972592  10.990000000   -0.000000224    0.0000000  -0.0         44
45    0.889715354   0.000001276   -0.000000189    0.0000000   0.0         45
46    0.900615535   0.000000805   -0.000000092   -0.0000000   0.0         46
47    0.904900598   0.000000621   -0.000000019    0.0000000   0.0         47
48    0.905118409  10.990000000    0.0           -0.0000000   0.0         48
49    0.492993197  10.990000000    0.000000000   -0.0000004  -0.0         49
50    0.504627306   0.000005789   -0.000000946   -0.0000003   0.0         50
51    0.634964686   0.000002820   -0.000001860   -0.0000000   0.0         51
52    0.755001169   0.000001307   -0.000001115   -0.0000000   0.0         52
53    0.778596830  10.990000000   -0.000000955   -0.0000000  -0.0         53
54    0.798710760   0.000000960   -0.000000797    0.0000000   0.0         54
55    0.843995013   0.000000583   -0.000000377   -0.0000000   0.0         55
56    0.861434767   0.000000445   -0.000000077   -0.0000000   0.0         56
57    0.862321354  10.990000000    0.0            0.0000000   0.0         57
58    0.012458879  10.990000000    0.000000000    0.0000002  -0.0         58
59    0.054689537   0.000003598   -0.000003373    0.0000001   0.0         59
60    0.457131234   0.000001174   -0.000004810   -0.0000000   0.0         60
61    0.686457602   0.000000353   -0.000001359    0.0000000   0.0         61
62    0.717198598  10.990000000   -0.000001474    0.0000000  -0.0         62
63    0.747857567   0.000000267   -0.000001199   -0.0000000   0.0         63
64    0.813939173   0.000000151   -0.000000533    0.0000000   0.0         64
65    0.838624250   0.000000113   -0.000000112   -0.0000000   0.0         65
66    0.839913476  10.990000000    0.0           -0.0000000   0.0         66
67    0.009859799   0.000000000   10.990000000   -0.0000165  -0.0000167   67
68    0.442863164  -0.000000000   10.990000000   -0.0000000  -0.0000000   68
69    0.682338930   0.0           10.990000000    0.0000000   0.0000000   69
70    0.744733978   0.000000000   10.990000000   -0.0000000   0.0         70
71    0.812191825   0.0           10.990000000    0.0000000   0.0         71
72    0.837314422   0.000000000   10.990000000   -0.0000000   0.0         72
```

TABLE F.2

CONFIGURATION ONE: CONCENTRATION RESULTS FOR DISPERSION COEFFICIENT=.01075 SQ.CM/S AND N=3

INITIAL VISCOSITY OF THE OIL= 100.000 CP
KD= 0.0107500SQ.CM/S
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT= 9
TIME=50,000 S
DT=50,000 S

COLLOCATION POINT	PRESSURE (PSI)	VELOCITY IN X DIRECTION CM/S	VELOCITY IN Y DIRECTION CM/S	CONCENTRATION		COLLOCATION POINT
				IMPLICIT METHOD	R-K METHOD	
1	1.828773953	-0.000000000	10.990000000	0.0086009	0.0083523	1
2	1.395770126	0.000000000	10.990000000	0.0000592	0.0000521	2
3	1.156293268	-0.000000000	10.990000000	-0.0000209	-0.0000338	3
4	1.093897308	-0.000000000	10.990000000	0.0000072	-0.0	4
5	1.026436607	0.000000000	10.990000000	-0.0000007	-0.0	5
6	1.001309415	-0.000000000	10.990000000	0.0000003	-0.0	6
7	1.826174900	10.990000000	-0.000000000	0.0086014	0.0083528	7
8	1.783944216	0.000003598	0.000003373	0.0075870	0.0073677	8
9	1.381502060	0.000001174	0.000004810	0.0000523	0.0000460	9
10	1.152174607	0.000000353	0.000001360	-0.0000185	-0.0000298	10
11	1.121433160	10.990000000	0.000001474	0.0002057	0.0001899	11
12	1.090773736	0.000000267	0.000001199	0.0000064	0.0	12
13	1.024689370	0.000000151	0.000000533	-0.0000006	0.0	13
14	1.000000000	0.000000113	0.000000112	0.0000003	0.0	14
15	0.998710335	10.990000000	0.000000000	-0.0000144	-0.0000146	15
16	1.345640597	10.990000000	0.000000000	0.0000614	0.0000543	16
17	1.334006465	0.000005789	0.000000946	0.0000542	0.0000479	17
18	1.203668649	0.000002820	0.000001860	0.0000007	0.0000004	18
19	1.083631182	0.000001307	0.000001115	-0.0000002	-0.0000002	19
20	1.060035134	10.990000000	0.000000955	0.0000014	0.0000012	20
21	1.039920776	0.000000960	0.000000797	0.0000001	0.0	21
22	0.994634706	0.000000583	0.000000377	-0.0000000	0.0	22
23	0.977193298	0.000000445	0.000000077	0.0000000	0.0	23
24	0.976306574	10.990000000	-0.000000000	-0.0000001	-0.0000001	24
25	0.995908837	10.990000000	0.000000000	-0.0000226	-0.0000354	25
26	0.995131576	0.000003179	0.000000067	-0.0000199	-0.0000312	26
27	0.981726407	0.000002717	0.000000254	-0.0000002	-0.0000002	27
28	0.959401065	0.000001736	0.000000271	0.0000001	0.0000002	28
29	0.953659550	10.990000000	0.000000224	-0.0000005	-0.0000008	29
30	0.948916384	0.000001276	0.000000189	-0.0000000	0.0	30
31	0.938014976	0.000000805	0.000000092	0.0000000	0.0	31
32	0.933729126	0.000000621	0.000000019	-0.0000000	0.0	32
33	0.933511264	10.990000000	-0.000000000	0.0000000	0.0000001	33

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34     0.919316895   0.000003576  10.990000000   0.0002049   0.0001889  34
35     0.919316689   0.000002726  10.990000000   0.0000013   0.0000012  35
36     0.919316243   0.000001739  10.990000000  -0.0000005  -0.0000007  36
37     0.919315898   0.000001291  10.990000000   0.0000002  -0.0        37
38     0.919315274   0.000000816  10.990000000  -0.0000000  -0.0        38
39     0.919314863   0.000000629  10.990000000   0.0000000  -0.0        39
40     0.842724974  10.990000000  -0.000000000   0.0000076  -0.0        40
41     0.843502213   0.000003179  -0.000000067   0.0000067   0.0        41
42     0.856906969   0.000002717  -0.000000254   0.0000001   0.0        42
43     0.879231393   0.000001736  -0.000000271  -0.0000000   0.0        43
44     0.884972592  10.990000000  -0.000000224   0.0000002  -0.0        44
45     0.889715354   0.000001276  -0.000000189   0.0000000   0.0        45
46     0.900615535   0.000000805  -0.000000092  -0.0000000   0.0        46
47     0.904900598   0.000000621  -0.000000019   0.0000000   0.0        47
48     0.905118409  10.990000000   0.0          -0.0000000   0.0        48
49     0.492993197  10.990000000   0.000000000  -0.0000010  -0.0        49
50     0.504627306   0.000005789  -0.000000946  -0.0000009   0.0        50
51     0.634964686   0.000002820  -0.000001860  -0.0000000   0.0        51
52     0.755001169   0.000001307  -0.000001115   0.0000000   0.0        52
53     0.778596830  10.990000000  -0.000000955  -0.0000000  -0.0        53
54     0.798710760   0.000000960  -0.000000797  -0.0000000   0.0        54
55     0.843995013   0.000000583  -0.000000377   0.0000000   0.0        55
56     0.861434767   0.000000445  -0.000000077  -0.0000000   0.0        56
57     0.862321354  10.990000000   0.0           0.0000000   0.0        57
58     0.012458879  10.990000000   0.000000000   0.0000005  -0.0        58
59     0.054689537   0.000003598  -0.000003373   0.0000004   0.0        59
60     0.457131234   0.000001174  -0.000004810   0.0000000   0.0        60
61     0.686457602   0.000000353  -0.000001359  -0.0000000   0.0        61
62     0.717198598  10.990000000  -0.000001474   0.0000000  -0.0        62
63     0.747857567   0.000000267  -0.000001199   0.0000000   0.0        63
64     0.813939173   0.000000151  -0.000000533  -0.0000000   0.0        64
65     0.838624250   0.000000113  -0.000000112   0.0000000   0.0        65
66     0.839913476  10.990000000   0.0          -0.0000000   0.0        66
67     0.009859799   0.000000000  10.990000000  -0.0000141  -0.0000145  67
68     0.442863164  -0.000000000  10.990000000  -0.0000001  -0.0000001  68
69     0.682338930   0.0          10.990000000   0.0000000  -0.0000001  69
70     0.744733978   0.000000000  10.990000000  -0.0000000   0.0        70
71     0.812191825   0.0          10.990000000   0.0000000   0.0        71
72     0.827314422   0.000000000  10.990000000  -0.0000000   0.0        72
```

TABLE F.3

CONFIGURATION ONE: CONCENTRATION RESULTS FOR DISPERSION COEFFICIENT=.0001075 SQ.CM/S AND N=5

INITIAL VISCOSITY OF THE OIL= 100.000 CP
KD= 0.0001075SQ.CM/S
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT= 25
TIME=50,000 S
DT=50,000 S

COLLOCATION POINT	PRESSURE (PSI)	VELOCITY IN X DIRECTION CM/S	VELOCITY IN Y DIRECTION CM/S	CONCENTRATION		COLLOCATION POINT
				IMPLICIT METHOD	R-K METHOD	
1	2.234607128	-0.000000000	10.990000000	0.0461453	0.0458244	1
2	1.737433350	0.000000000	10.990000000	0.0003190	0.0003498	2
3	1.376337514	-0.000000000	10.990000000	-0.0000300	-0.0000397	3
4	1.216296452	-0.000000000	10.990000000	0.0000174	0.0000238	4
5	1.136850047	-0.000000000	10.990000000	-0.0000062	-0.0000085	5
6	1.110997583	-0.000000000	10.990000000	0.0000020	-0.0	6
7	1.067341590	0.000000000	10.990000000	-0.0000003	-0.0	7
8	1.026412695	-0.000000000	10.990000000	0.0000001	-0.0	8
9	1.005394978	0.000000000	10.990000000	-0.0000000	-0.0	9
10	1.000220212	-0.000000000	10.990000000	0.0000000	-0.0	10
11	2.234165861	10.990000000	-0.000000000	0.0461482	0.0458278	11
12	2.190994862	0.000008310	0.000008218	0.0412828	0.0410013	12
13	1.725676028	0.000002306	0.000011613	0.0002854	0.0003129	13
14	1.374494719	0.000000389	0.000002910	-0.0000268	-0.0000355	14
15	1.215013063	0.000000262	0.000002984	0.0000155	0.0000213	15
16	1.136410532	0.000000097	0.000001210	-0.0000055	-0.0000076	16
17	1.124150168	10.990000000	0.000001501	0.0003819	0.0003756	17
18	1.110398502	0.000000124	0.000001373	0.0000018	0.0	18
19	1.066915711	0.000000089	0.000000966	-0.0000003	0.0	19
20	1.026116075	0.000000062	0.000000544	0.0000001	0.0	20
21	1.005159837	0.000000049	0.000000231	-0.0000000	0.0	21
22	1.000000000	0.000000046	0.000000046	0.0000000	0.0	22
23	0.999778939	10.990000000	0.000000000	-0.0000122	-0.0000121	23
24	1.726776296	10.990000000	0.000000000	0.0003311	0.0003632	24
25	1.715459344	0.000012065	0.000002214	0.0002962	0.0003248	25
26	1.560109222	0.000005339	0.000004887	0.0000019	0.0000015	26
27	1.331611033	0.000001817	0.000003230	-0.0000002	-0.0000002	27
28	1.189845828	0.000001019	0.000002081	0.0000001	0.0000001	28
29	1.123167673	0.000000624	0.000001478	-0.0000000	-0.0000000	29
30	1.109674172	10.990000000	0.000001357	0.0000027	0.0000030	30
31	1.097194358	0.000000567	0.000001251	0.0000000	0.0	31
32	1.057230470	0.000000422	0.000000894	-0.0000000	0.0	32
33	1.019322084	0.000000297	0.000000507	0.0000000	0.0	33

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```
34   0.999763864   0.000000236   0.000000216  -0.0000000   0.0         34
35   0.994943017   0.000000222   0.000000043   0.0000000   0.0         35
36   0.994737782  10.990000000   0.000000000  -0.0000001  -0.0000001   36
37   1.326227252  10.990000000  -0.000000000  -0.0000395  -0.0000524   37
38   1.324835028   0.000003895   0.000000294  -0.0000354  -0.0000469   38
39   1.292556261   0.000004206   0.000001358  -0.0000003  -0.0000003   39
40   1.199988026   0.000002731   0.000001769   0.0000000   0.0000000   40
41   1.114299050   0.000001663   0.000001317  -0.0000000  -0.0000000   41
42   1.070058200   0.000001244   0.000001047   0.0000000   0.0000000   42
43   1.060406743  10.990000000   0.000000965  -0.0000003  -0.0000004   43
44   1.051507648   0.000001062   0.000000895  -0.0000000   0.0         44
45   1.022612394   0.000000815   0.000000653   0.0000000   0.0         45
46   0.994660298   0.000000586   0.000000377  -0.0000000   0.0         46
47   0.980066789   0.000000469   0.000000162   0.0000000   0.0         47
48   0.976455723   0.000000441   0.000000032  -0.0000000   0.0         48
49   0.976301766  10.990000000   0.0           0.0000000   0.0000000   49
50   1.096821423  10.990000000   0.000000000   0.0000258   0.0000354   50
51   1.096040164   0.000004532   0.000000157   0.0000231   0.0000316   51
52   1.082352697   0.000003593   0.000000517   0.0000001   0.0000001   52
53   1.048646139   0.000002736   0.000000663  -0.0000000  -0.0000000   53
54   1.013319300   0.000001960   0.000000595   0.0000000   0.0000000   54
55   0.993121893   0.000001515   0.000000479  -0.0000000  -0.0000000   55
56   0.988671966  10.990000000   0.000000451   0.0000002   0.0000003   56
57   0.984485416   0.000001339   0.000000423   0.0000000   0.0         57
58   0.970635880   0.000001048   0.000000317  -0.0000000   0.0         58
59   0.956958608   0.000000766   0.000000186   0.0000000   0.0         59
60   0.949735906   0.000000618   0.000000080  -0.0000000   0.0         60
61   0.947941227   0.000000581   0.000000016   0.0000000   0.0         61
62   0.947864928  10.990000000   0.0          -0.0000000  -0.0000000   62
63   0.951092704  10.990000000  -0.000000000  -0.0000137  -0.0000170   63
64   0.951268170   0.000003202  -0.000000031  -0.0000123  -0.0000152   64
65   0.951620697   0.000003382   0.000000041  -0.0000001  -0.0000001   65
66   0.945670817   0.000002756   0.000000143   0.0000000   0.0000000   66
67   0.938572534   0.000002009   0.000000111  -0.0000000  -0.0000000   67
68   0.934560825   0.000001591   0.000000102   0.0000000   0.0000000   68
69   0.933522680  10.990000000   0.000000092  -0.0000001  -0.0000001   69
70   0.932766954   0.000001403   0.000000086  -0.0000000   0.0         70
71   0.929935330   0.000001108   0.000000065   0.0000000   0.0         71
72   0.927118607   0.000000814   0.000000038  -0.0000000   0.0         72
73   0.925625095   0.000000658   0.000000017   0.0000000   0.0         73
74   0.925253672   0.000000619   0.000000003  -0.0000000   0.0         74
75   0.925237764  10.990000000   0.0           0.0000000   0.0000000   75
76   0.919314275   0.000003623  10.990000000   0.0003779   0.0003699   76
77   0.919314096   0.000003374  10.990000000   0.0000026   0.0000029   77
78   0.919313838   0.000002745  10.990000000  -0.0000002  -0.0000003   78
79   0.919313490   0.000002013  10.990000000   0.0000001   0.0000002   79
80   0.919313277   0.000001591  10.990000000  -0.0000001  -0.0000001   80
81   0.919313086   0.000001407  10.990000000   0.0000000  -0.0         81
82   0.919312565   0.000001110  10.990000000  -0.0000000  -0.0         82
83   0.919311957   0.000000816  10.990000000   0.0000000  -0.0         83
84   0.919311559   0.000000660  10.990000000  -0.0000000  -0.0         84
85   0.919311548   0.000000621  10.990000000   0.0000000  -0.0         85
86   0.887535858  10.990000000   0.000000000   0.0000037  -0.0         86
87   0.887360300   0.000003202   0.000000031   0.0000033   0.0         87
88   0.887007407   0.000003382  -0.000000041   0.0000000   0.0         88
89   0.892956771   0.000002756  -0.000000143  -0.0000000   0.0         89
90   0.900054357   0.000002009  -0.000000111   0.0000000   0.0         90
91   0.904065639   0.000001591  -0.000000102  -0.0000000   0.0         91
92   0.905003683  10.990000000  -0.000000092   0.0000000  -0.0         92
93   0.905859127   0.000001403  -0.000000086   0.0000000   0.0         93
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CONTINUED

 94        0.908689709     0.000001108    -0.000000065    -0.0000000    0.0           94
 95        0.911505210     0.000000814    -0.000000038     0.0000000    0.0           95
 96        0.912997921     0.000000658    -0.000000017    -0.0000000    0.0           96
 97        0.913369357     0.000000619    -0.000000003     0.0000000    0.0           97
 98        0.913385344    10.990000000     0.0            -0.0000000    0.0           98
 99        0.741606887    10.990000000     0.000000000    -0.0000012   -0.0           99
100        0.742588053     0.000004532    -0.000000157    -0.0000011    0.0          100
101        0.756275154     0.000003593    -0.000000517    -0.0000000    0.0          101
102        0.789981189     0.000002736    -0.000000663     0.0000000    0.0          102
103        0.825307313     0.000001960    -0.000000595    -0.0000000    0.0          103
104        0.845504271     0.000001515    -0.000000479     0.0000000    0.0          104
105        0.849954089    10.990000000    -0.000000451    -0.0000000   -0.0          105
106        0.854140350     0.000001339    -0.000000423    -0.0000000    0.0          106
107        0.867988797     0.000001048    -0.000000317     0.0000000    0.0          107
108        0.881664743     0.000000766    -0.000000186    -0.0000000    0.0          108
109        0.888886499     0.000000618    -0.000000080     0.0000000    0.0          109
110        0.890681082     0.000000581    -0.000000016    -0.0000000    0.0          110
111        0.890757450    10.990000000    -0.000000000     0.0000000    0.0          111
112        0.512400976    10.990000000     0.000000000     0.0000005   -0.0          112
113        0.513793108     0.000003895    -0.000000294     0.0000005    0.0          113
114        0.546071503     0.000004206    -0.000001358     0.0000000    0.0          114
115        0.638639194     0.000002731    -0.000001769    -0.0000000    0.0          115
116        0.724327403     0.000001663    -0.000001317     0.0000000    0.0          116
117        0.768567732     0.000001244    -0.000001047     0.0000000    0.0          117
118        0.778219053    10.990000000    -0.000000965     0.0000000   -0.0          118
119        0.787117828     0.000001062    -0.000000895     0.0000000    0.0          119
120        0.816011833     0.000000815    -0.000000653     0.0000000    0.0          120
121        0.843962176     0.000000586    -0.000000377    -0.0000000    0.0          121
122        0.858554142     0.000000469    -0.000000162    -0.0000000    0.0          122
123        0.862164920     0.000000441    -0.000000032     0.0000000    0.0          123
124        0.862318942    10.990000000     0.000000000    -0.0000000    0.0          124
125        0.111851979    10.990000000     0.000000000    -0.0000010   -0.0          125
126        0.123168839     0.000012065    -0.000002214    -0.0000009    0.0          126
127        0.276518585     0.000005339    -0.000004887    -0.0000000    0.0          127
128        0.506816208     0.000001817    -0.000003230    -0.0000000    0.0          128
129        0.648780591     0.000001019    -0.000002081     0.0000000    0.0          129
130        0.715438145     0.000000624    -0.000001474    -0.0000000    0.0          130
131        0.728951484    10.950000000    -0.000001357    -0.0000000   -0.0          131
132        0.741430946     0.000000567    -0.000001251     0.0000000    0.0          132
133        0.781393369     0.000000422    -0.000000894    -0.0000000    0.0          133
134        0.819299371     0.000000297    -0.000000507     0.0000000    0.0          134
135        0.838854326     0.000000236    -0.000000216    -0.0000000    0.0          135
136        0.843673269     0.000000222    -0.000000043    -0.0000000    0.0          136
137        0.843878438    10.990000000    -0.000000000     0.0000000    0.0          137
138       -0.395537344    10.990000000     0.000000000     0.0000006   -0.0          138
139       -0.352366438     0.000008310    -0.000008218     0.0000005    0.0          139
140        0.112952019     0.000002306    -0.000011613    -0.0000000    0.0          140
141        0.464132755     0.000000389    -0.000002910     0.0000000    0.0          141
142        0.623613570     0.000000262    -0.000002984    -0.0000000    0.0          142
143        0.702215471     0.000000097    -0.000001210     0.0000000    0.0          143
144        0.714475660    10.990000000    -0.000001501     0.0000000   -0.0          144
145        0.726226996     0.000000124    -0.000001373    -0.0000000    0.0          145
146        0.771708184     0.000000089    -0.000000966     0.0000000    0.0          146
147        0.812505230     0.000000062    -0.000000544    -0.0000000    0.0          147
148        0.833456586     0.000000049    -0.000000231     0.0000000    0.0          148
149        0.838610392     0.000000046    -0.000000046     0.0000000    0.0          149
150        0.838830961    10.990000000     0.000000000    -0.0000000    0.0          150
151       -0.395978617    -0.000000000    10.990000000    -0.0000112   -0.0000119   151
152        0.101194783     0.0            10.990000000    -0.0000001   -0.0000001   152
153        0.462290047    -0.000000000    10.990000000     0.0000000    0.0000000   153
154        0.622330226     0.0            10.990000000    -0.0000000   -0.0000000   154
155        0.701776040     0.0            10.990000000     0.0000000    0.0000000   155
156        0.727628001     0.0            10.990000000    -0.0000000    0.0          156
157        0.771282381     0.0            10.990000000     0.0000000    0.0          157
158        0.812208681     0.0            10.990000000    -0.0000000    0.0          158
159        0.833221384     0.000000000    10.990000000     0.0000000    0.0          159
160        0.838389694     0.000000000    10.990000000    -0.0000000    0.0          160
```

TABLE F.4

CONFIGURATION ONE: CONCENTRATION RESULTS FOR DISPERSION COEFFICIENT=.01075 SQ.CM/S AND N=5

INITIAL VISCOSITY OF THE OIL=   100.000 CP
KD=   0.0107500SQ.CM/S
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT= 25
TIME=50,000 S
DT=50,000 S

COLLOCATION POINT	PRESSURE (PSI)	VELOCITY IN X DIRECTION CM/S	VELOCITY IN Y DIRECTION CM/S	CONCENTRATION		COLLOCATION POINT
				IMPLICIT METHOD	R-K METHOD	
1	2.234607128	-0.000000000	10.990000000	0.0430028	0.0367040	1
2	1.737433350	0.000000000	10.990000000	0.0011293	0.0009224	2
3	1.376337514	-0.000000000	10.990000000	-0.0001575	-0.0001864	3
4	1.216296492	-0.000000000	10.990000000	0.0000958	0.0001348	4
5	1.136850047	-0.000000000	10.990000000	-0.0001136	-0.0002525	5
6	1.110997583	-0.000000000	10.990000000	0.0000464	-0.0	6
7	1.067341590	0.000000000	10.990000000	-0.0000024	-0.0	7
8	1.026412695	-0.000000000	10.990000000	0.0000010	-0.0	8
9	1.005394978	0.000000000	10.990000000	-0.0000007	-0.0	9
10	1.000220212	-0.000000000	10.990000000	0.0000006	-0.0	10
11	2.234165861	10.990000000	-0.000000000	0.0430052	0.0367067	11
12	2.190994862	0.000008310	0.000008218	0.0365935	0.0329442	12
13	1.725676028	0.000002306	0.000011613	0.0010166	0.0008283	13
14	1.374494719	0.000000389	0.000002910	-0.0001418	-0.0001675	14
15	1.215013063	0.000000262	0.000002984	0.0000862	0.0001210	15
16	1.136410532	0.000000097	0.000001210	-0.0001022	-0.0002267	16
17	1.124150168	10.990000000	0.000001501	0.0002993	0.0001529	17
18	1.110398502	0.000000124	0.000001373	0.0000417	0.0	18
19	1.066915711	0.000000089	0.000000966	-0.0000022	0.0	19
20	1.026116075	0.000000062	0.000000544	0.0000009	0.0	20
21	1.005159837	0.000000049	0.000000231	-0.0000007	0.0	21
22	1.000000000	0.000000046	0.000000046	0.0000006	0.0	22
23	0.999778939	10.990000000	0.000000000	-0.0000064	-0.0000049	23
24	1.726776296	10.990000000	0.000000000	0.0011397	0.0000329	24
25	1.715459344	0.000012065	0.000002214	0.0010258	0.0008377	25
26	1.560109222	0.000005339	0.000004887	0.0000463	0.0000234	26
27	1.331811033	0.000001817	0.000003230	-0.0000064	-0.0000050	27
28	1.189845828	0.000001019	0.000002081	0.0000037	0.0000034	28
29	1.123167673	0.000000624	0.000001474	-0.0000043	-0.0000063	29
30	1.109674172	10.990000000	0.000001357	0.0000067	0.0000035	30
31	1.097194358	0.000000567	0.000001251	0.0000016	0.0	31
32	1.057230470	0.000000422	0.000000894	-0.0000001	0.0	32
33	1.019322084	0.000000297	0.000000507	0.0000000	0.0	33

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```
34         0.999763864    0.000000236    0.000000216   -0.0000000    0.0          34
35         0.994943017    0.000000222    0.000000043    0.0000000    0.0          35
36         0.994737782   10.990000000    0.000000000   -0.0000001   -0.0000001    36
37         1.326227252   10.990000000   -0.000000000   -0.0001652   -0.0001964    37
38         1.324835028    0.000003895    0.000000294   -0.0001487   -0.0001764    38
39         1.292556261    0.000004206    0.000001358   -0.0000067   -0.0000051    39
40         1.199988026    0.000002731    0.000001769    0.0000009    0.0000011    40
41         1.114299050    0.000001663    0.000001317   -0.0000005   -0.0000008    41
42         1.070058200    0.000001244    0.000001047    0.0000006    0.0000014    42
43         1.060406743   10.990000000    0.000000965   -0.0000010   -0.0000007    43
44         1.051507648    0.000001062    0.000000895   -0.0000002    0.0          44
45         1.022612394    0.000000815    0.000000653    0.0000000    0.0          45
46         0.994660298    0.000000586    0.000000377   -0.0000000    0.0          46
47         0.980066789    0.000000469    0.000000162    0.0000000    0.0          47
48         0.976455723    0.000000441    0.000000032   -0.0000000    0.0          48
49         0.976301766   10.990000000    0.0            0.0000000    0.0000000    49
50         1.096821423   10.990000000    0.000000000    0.0001025    0.0001438    50
51         1.096040164    0.000004532    0.000000157    0.0000922    0.0001291    51
52         1.082352697    0.000003593    0.000000517    0.0000039    0.0000036    52
53         1.048646139    0.000002736    0.000000663   -0.0000006   -0.0000008    53
54         1.013319300    0.000001960    0.000000595    0.0000003    0.0000005    54
55         0.993121893    0.000001515    0.000000479   -0.0000004   -0.0000010    55
56         0.988671966   10.990000000    0.000000451    0.0000006    0.0000005    56
57         0.984485416    0.000001339    0.000000423    0.0000002    0.0          57
58         0.970635880    0.000001048    0.000000317   -0.0000000    0.0          58
59         0.956958608    0.000000766    0.000000186    0.0000000    0.0          59
60         0.949735906    0.000000618    0.000000080   -0.0000000    0.0          60
61         0.947941227    0.000000581    0.000000016    0.0000000    0.0          61
62         0.947864928   10.990000000    0.0           -0.0000000   -0.0000000    62
63         0.951092704   10.990000000   -0.000000000   -0.0001188   -0.0002586    63
64         0.951268170    0.000003202   -0.000000031   -0.0001069   -0.0002322    64
65         0.951620697    0.000003382    0.000000041   -0.0000045   -0.0000063    65
66         0.945670817    0.000002756    0.000000143    0.0000006    0.0000013    66
67         0.938572534    0.000002009    0.000000111   -0.0000004   -0.0000010    67
68         0.934560825    0.000001591    0.000000102    0.0000005    0.0000018    68
69         0.933622680   10.990000000    0.000000092   -0.0000007   -0.0000009    69
70         0.932766954    0.000001403    0.000000086   -0.0000002    0.0          70
71         0.929935330    0.000001108    0.000000065    0.0000000    0.0          71
72         0.927118607    0.000000614    0.000000038   -0.0000000    0.0          72
73         0.925625095    0.000000658    0.000000017    0.0000000    0.0          73
74         0.925253672    0.000000619    0.000000003   -0.0000000    0.0          74
75         0.925237764   10.990000000    0.0            0.0000000    0.0000000    75
76         0.919314279    0.000003623   10.990000000    0.0002961    0.0001487    76
77         0.919314096    0.000003374   10.990000000    0.0000065    0.0000034    77
78         0.919313838    0.000002745   10.990000000   -0.0000009   -0.0000006    78
79         0.919313450    0.000002013   10.990000000    0.0000006    0.0000005    79
80         0.919313277    0.000001591   10.990000000   -0.0000007   -0.0000009    80
81         0.919313086    0.000001407   10.990000000    0.0000003   -0.0          81
82         0.919312565    0.000001110   10.990000000   -0.0000000   -0.0          82
83         0.919311957    0.000000816   10.990000000    0.0000000   -0.0          83
84         0.919311559    0.000000660   10.990000000   -0.0000000   -0.0          84
85         0.919311548    0.000000621   10.990000000    0.0000000   -0.0          85
86         0.887535858   10.990000000    0.000000000    0.0000468   -0.0          86
87         0.887360300    0.000003202    0.000000031    0.0000421    0.0          87
88         0.887007407    0.000003382   -0.000000041    0.0000016    0.0          88
89         0.892956771    0.000002756   -0.000000143   -0.0000002    0.0          89
90         0.900054357    0.000002009   -0.000000111    0.0000001    0.0          90
91         0.904065639    0.000001591   -0.000000102   -0.0000002    0.0          91
92         0.905003683   10.990000000   -0.000000092    0.0000003   -0.0          92
93         0.905859127    0.000001403   -0.000000086    0.0000001    0.0          93
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 94        0.908689709    0.000001108   -0.000000065   -0.0000000    0.0          94
 95        0.911505210    0.000000814   -0.000000038    0.0000000    0.0          95
 96        0.912997921    0.000000658   -0.000000017   -0.0000000    0.0          96
 97        0.913369357    0.000000619   -0.000000003    0.0000000    0.0          97
 98        0.913385344   10.990000000    0.0           -0.0000000    0.0          98
 99        0.741806887   10.990000000    0.000000000   -0.0000029   -0.0          99
100        0.742588053    0.000004532   -0.000000157   -0.0000026    0.0         100
101        0.756275154    0.000003593   -0.000000517   -0.0000001    0.0         101
102        0.789981189    0.000002736   -0.000000663    0.0000000    0.0         102
103        0.825307313    0.000001960   -0.000000595   -0.0000000    0.0         103
104        0.845504271    0.000001515   -0.000000479    0.0000000    0.0         104
105        0.849954089   10.990000000   -0.000000451   -0.0000000   -0.0         105
106        0.854140350    0.000001339   -0.000000423   -0.0000000    0.0         106
107        0.867988797    0.000001048   -0.000000317    0.0000000    0.0         107
108        0.881664743    0.000000766   -0.000000186   -0.0000000    0.0         108
109        0.888886499    0.000000618   -0.000000080    0.0000000    0.0         109
110        0.890681082    0.000000581   -0.000000016   -0.0000000    0.0         110
111        0.890757450   10.990000000   -0.000000000    0.0000000    0.0         111
112        0.512400976   10.990000000    0.000000000    0.0000012   -0.0         112
113        0.513793108    0.000003895   -0.000000294    0.0000011    0.0         113
114        0.546071503    0.000004206   -0.000001358    0.0000000    0.0         114
115        0.638639194    0.000002731   -0.000001769   -0.0000000    0.0         115
116        0.724327403    0.000001663   -0.000001317    0.0000000    0.0         116
117        0.768567732    0.000001244   -0.000001047   -0.0000000    0.0         117
118        0.778219053   10.990000000   -0.000000965    0.0000000   -0.0         118
119        0.787117828    0.000001062   -0.000000855    0.0000000    0.0         119
120        0.816011833    0.000000815   -0.000000653   -0.0000000    0.0         120
121        0.843962176    0.000000586   -0.000000377    0.0000000    0.0         121
122        0.858554142    0.000000469   -0.000000162   -0.0000000    0.0         122
123        0.862164920    0.000000441   -0.000000032    0.0000000    0.0         123
124        0.862318942   10.990000000    0.000000000   -0.0000000    0.0         124
125        0.111851979   10.990000000    0.000000000   -0.0000013   -0.0         125
126        0.123168839    0.000012065   -0.000002214   -0.0000011    0.0         126
127        0.278518585    0.000005339   -0.000004887   -0.0000000    0.0         127
128        0.506816208    0.000001817   -0.000003230    0.0000000    0.0         128
129        0.648780591    0.000001019   -0.000002081   -0.0000000    0.0         129
130        0.715438145    0.000000624   -0.000001474    0.0000000    0.0         130
131        0.728951484   10.990000000   -0.000001357   -0.0000000   -0.0         131
132        0.741430946    0.000000567   -0.000001251   -0.0000000    0.0         132
133        0.781393369    0.000000422   -0.000000894    0.0000000    0.0         133
134        0.819299371    0.000000297   -0.000000507   -0.0000000    0.0         134
135        0.838854326    0.000000236   -0.000000216    0.0000000    0.0         135
136        0.843673269    0.000000222   -0.000000043   -0.0000000    0.0         136
137        0.843878438   10.990000000   -0.000000000    0.0000000    0.0         137
138       -0.395537344   10.990000000    0.000000000    0.0000009   -0.0         138
139       -0.352366438    0.000008310   -0.000008218    0.0000008    0.0         139
140        0.112952019    0.000002306   -0.000011613    0.0000000    0.0         140
141        0.464132755    0.000000389   -0.000002910   -0.0000000    0.0         141
142        0.623613570    0.000000262   -0.000002984    0.0000000    0.0         142
143        0.702215471    0.000000097   -0.000001210   -0.0000000    0.0         143
144        0.714475660   10.990000000   -0.000001501    0.0000000   -0.0         144
145        0.728226996    0.000000124   -0.000001373    0.0000000    0.0         145
146        0.771708184    0.000000089   -0.000000966   -0.0000000    0.0         146
147        0.812505230    0.000000062   -0.000000544    0.0000000    0.0         147
148        0.833456586    0.000000049   -0.000000231   -0.0000000    0.0         148
149        0.836610392    0.000000046   -0.000000046    0.0000000    0.0         149
150        0.838830961   10.990000000    0.000000000   -0.0000000    0.0         150
151       -0.395978617   -0.000000000   10.990000000   -0.0000059   -0.0000048   151
152        0.101194783    0.0           10.990000000   -0.0000001   -0.0000001   152
153        0.462290047   -0.000000000   10.990000000    0.0000000    0.0000000   153
154        0.622330226    0.0           10.990000000   -0.0000000   -0.0000000   154
155        0.701776040    0.0           10.990000000    0.0000000    0.0000000   155
156        0.727628001    0.0           10.990000000   -0.0000000    0.0         156
157        0.771282381    0.0           10.990000000    0.0000000    0.0         157
158        0.812208681    0.0           10.990000000   -0.0000000    0.0         158
159        0.833221384    0.000000000   10.990000000    0.0000000    0.0         159
160        0.838389694    0.000000000   10.990000000   -0.0000000    0.0         160
```

TABLE F.5

CONFIGURATION TWO: CONCENTRATION RESULTS FOR DISPERSION COEFFICIENT=.0001075 SQ.CM/S AND N=3

INITIAL VISCOSITY OF THE OIL= 100.000 CP
KD= 0.0001075SQ.CM/S
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT= 9
TIME=50,000 S
DT=50,000 S

COLLOCATION POINT	PRESSURE (PSI)	VELOCITY IN X DIRECTION CM/S	VELOCITY IN Y DIRECTION CM/S	CONCENTRATION		COLLOCATION POINT
				IMPLICIT METHOD	R-K METHOD	
1	1.988869925	-0.000000000	10.990000000	0.0087392	0.0087289	1
2	1.533457832	0.000000000	10.990000000	0.0000125	0.0000128	2
3	1.251182955	0.000000000	10.990000000	-0.0000022	-0.0000022	3
4	1.160393302	0.000000000	10.990000000	0.0000005	-0.0	4
5	1.050134430	0.000000000	10.990000000	-0.0000001	-0.0	5
6	1.002598610	-0.000000000	10.990000000	0.0000000	-0.0	6
7	1.988869953	10.990000000	-0.000000000	0.0087392	0.0087289	7
8	1.945329422	0.000003486	0.000003486	0.0076994	0.0076904	8
9	1.520076363	0.000001097	0.000005256	0.0000110	0.0000113	9
10	1.247282129	0.000000334	0.000001980	-0.0000019	-0.0000019	10
11	1.202126004	10.990000000	0.000002102	0.0002189	0.0002182	11
12	1.157051901	0.000000286	0.000001820	0.0000004	0.0	12
13	1.047500608	0.000000228	0.000000979	-0.0000001	0.0	13
14	1.000000000	0.000000225	0.000000225	0.0000000	0.0	14
15	0.997400451	10.990000000	0.000000000	-0.0000167	-0.0000168	15
16	1.533458380	10.990000000	0.000000000	0.0000125	0.0000128	16
17	1.520076882	0.000005256	0.000001097	0.0000110	0.0000113	17
18	1.355682893	0.000002443	0.000002443	0.0000000	0.0000000	18
19	1.183024121	0.000001215	0.000001920	-0.0000000	-0.0000000	19
20	1.140728013	10.990000000	0.000001771	0.0000003	0.0000003	20
21	1.101913591	0.000001052	0.000001602	0.0000000	0.0	21
22	1.000006148	0.000000960	0.000000960	-0.0000000	0.0	22
23	0.952508463	0.000000979	0.000000228	0.0000000	0.0	23
24	0.949874364	10.990000000	0.000000000	-0.0000000	-0.0000000	24
25	1.251184709	10.990000000	0.000000000	-0.0000022	-0.0000022	25
26	1.247283846	0.000001980	0.000000334	-0.0000019	-0.0000019	26
27	1.183025236	0.000001920	0.000001215	-0.0000000	-0.0000000	27
28	1.069694732	0.000001547	0.000001547	0.0000000	0.0000000	28
29	1.034352484	10.990000000	0.000001515	-0.0000001	-0.0000001	29
30	1.000008603	0.000001465	0.000001465	-0.0000000	0.0	30
31	0.898101992	0.000001602	0.000001052	0.0000000	0.0	31
32	0.842962680	0.000001820	0.000000286	-0.0000000	0.0	32
33	0.839621215	10.990000000	0.000000000	0.0000000	0.0000000	33

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34        1.202128214    0.000002102   10.990000000    0.0002188    0.0002182   34
35        1.140729552    0.000001771   10.990000000    0.0000003    0.0000003   35
36        1.034352859    0.000001515   10.990000000   -0.0000001   -0.0000001   36
37        0.965665180    0.000001515   10.990000000    0.0000000   -0.0         37
38        0.859288295    0.000001771   10.990000000   -0.0000000   -0.0         38
39        0.797889409    0.000002102   10.990000000    0.0000000   -0.0         39
40        1.160396049   10.990000000   -0.000000000    0.0000005   -0.0         40
41        1.157054605    0.000001820    0.000000286    0.0000004    0.0         41
42        1.101915593    0.000001602    0.000001052    0.0000000    0.0         42
43        1.000009365    0.000001465    0.000001465   -0.0000000    0.0         43
44        0.965665556   10.990000000    0.000001515    0.0000000   -0.0         44
45        0.930323350    0.000001547    0.000001547    0.0000000    0.0         45
46        0.616992827    0.000001920    0.000001215   -0.0000000    0.0         46
47        0.752734110    0.000001980    0.000000334    0.0000000    0.0         47
48        0.748833237   10.990000000    0.0           -0.0000000    0.0         48
49        1.050140264   10.990000000    0.000000000   -0.0000001   -0.0         49
50        1.047506295    0.000000979    0.000000228   -0.0000001    0.0         50
51        1.000010081    0.000000960    0.000000960   -0.0000000    0.0         51
52        0.898103994    0.000001052    0.000001602    0.0000000    0.0         52
53        0.859289834   10.990000000    0.000001771   -0.0000000   -0.0         53
54        0.616993942    0.000001215    0.000001920   -0.0000000    0.0         54
55        0.640335622    0.000002443    0.000002443    0.0000000    0.0         55
56        0.479941778    0.000005256    0.000001097   -0.0000000    0.0         56
57        0.466560287   10.990000000    0.0            0.0000000    0.0         57
58        1.002609424   10.990000000    0.000000000    0.0000000   -0.0         58
59        1.000010338    0.000000225    0.000000225    0.0000000    0.0         59
60        0.952514151    0.000000228    0.000000979    0.0000000    0.0         60
61        0.842965384    0.000000286    0.000001820   -0.0000000    0.0         61
62        0.797891619   10.990000000    0.000002102    0.0000000   -0.0         62
63        0.752735827    0.000000334    0.000001980    0.0000000    0.0         63
64        0.479942297    0.000001097    0.000005256   -0.0000000    0.0         64
65        0.054689489    0.000003486    0.000003486    0.0000000    0.0         65
66        0.011148970   10.990000000   -0.000000000   -0.0000000    0.0         66
67        0.997411265   -0.000000000   10.990000000   -0.0000167   -0.0000168   67
68        0.949880198    0.000000000   10.990000000   -0.0000000   -0.0000000   68
69        0.839623961   -0.000000000   10.990000000    0.0000000    0.0000000   69
70        0.748834991   -0.000000000   10.990000000   -0.0000000    0.0         70
71        0.466560835    0.000000000   10.990000000    0.0000000    0.0         71
72        0.011146999   -0.000000000   10.990000000   -0.0000000    0.0         72
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TABLE F.6

CONFIGURATION TWO: CONCENTRATION RESULTS FOR DISPERSION COEFFICIENT=.01075 SQ.CM/S AND N=3

```
INITIAL VISCOSITY OF THE OIL=   100.000 CP
KD=     0.0107500SQ.CM/S
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT=  9
TIME=50,000 S
DT=50,000 S
```

COLLOCATION POINT	PRESSURE (PSI)	VELOCITY IN X DIRECTION CM/S	VELOCITY IN Y DIRECTION CM/S	CONCENTRATION		COLLOCATION POINT
				IMPLICIT METHOD	R-K METHOD	
1	1.988869925	-0.000000000	10.990000000	0.0086012	0.0083526	1
2	1.533457832	0.000000000	10.990000000	0.0000602	0.0000531	2
3	1.251182955	0.000000000	10.990000000	-0.0000215	-0.0000343	3
4	1.160393302	0.000000000	10.990000000	0.0000073	-0.0	4
5	1.050134430	0.000000000	10.990000000	-0.0000007	-0.0	5
6	1.002598610	-0.000000000	10.990000000	0.0000002	-0.0	6
7	1.988869953	10.990000000	-0.000000000	0.0086012	0.0063526	7
8	1.945329422	0.000003486	0.000003486	0.0075870	0.0073677	8
9	1.520076363	0.000001097	0.000005256	0.0000532	0.0000469	9
10	1.247282129	0.000000334	0.000001980	-0.0000190	-0.0000303	10
11	1.202126004	10.990000000	0.000002102	0.0002054	0.0001896	11
12	1.157051901	0.000000286	0.000001820	0.0000065	0.0	12
13	1.047500608	0.000000228	0.000000979	-0.0000007	0.0	13
14	1.000000000	0.000000225	0.000000225	0.0000003	0.0	14
15	0.997400451	10.990000000	0.000000000	-0.0000144	-0.0000146	15
16	1.533458380	10.990000000	0.000000000	0.0000602	0.0000531	16
17	1.520076882	0.000005256	0.000001097	0.0000532	0.0000469	17
18	1.359682893	0.000002443	0.000002443	0.0000007	0.0000004	18
19	1.183024121	0.000001215	0.000001920	-0.0000002	-0.0000002	19
20	1.140728013	10.990000000	0.000001771	0.0000014	0.0000012	20
21	1.101913591	0.000001052	0.000001602	0.0000001	0.0	21
22	1.000006148	0.000000960	0.000000960	-0.0000000	0.0	22
23	0.952508463	0.000000979	0.000000228	0.0000000	0.0	23
24	0.949874364	10.990000000	0.000000000	-0.0000001	-0.0000001	24
25	1.251184709	10.990000000	0.000000000	-0.0000215	-0.0000343	25
26	1.247283846	0.000001980	0.000000334	-0.0000190	-0.0000303	26
27	1.183025236	0.000001920	0.000001215	-0.0000002	-0.0000002	27
28	1.069694732	0.000001547	0.000001547	0.0000001	0.0000002	28
29	1.034352484	10.990000000	0.000001515	-0.0000005	-0.0000008	29
30	1.000008603	0.000001465	0.000001465	-0.0000000	0.0	30
31	0.898101992	0.000001602	0.000001052	0.0000000	0.0	31
32	0.842962680	0.000001820	0.000000286	-0.0000000	0.0	32
33	0.839621215	10.990000000	0.000000000	0.0000000	0.0000001	33

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34      1.202128214   0.000002102  10.990000000   0.0002054   0.0001896  34
35      1.140729552   0.000001771  10.950000000   0.0000014   0.0000012  35
36      1.034352859   0.000001515  10.990000000  -0.0000005  -0.0000008  36
37      0.965665180   0.000001515  10.990000000   0.0000002  -0.0        37
38      0.859288255   0.000001771  10.990000000  -0.0000000  -0.0        38
39      0.757889409   0.000002102  10.990000000   0.0000000  -0.0        39
40      1.160396049  10.990000000  -0.000000000   0.0000073  -0.0        40
41      1.157054605   0.000001820   0.000000286   0.0000065   0.0        41
42      1.101915593   0.000001602   0.000001052   0.0000001   0.0        42
43      1.000009365   0.000001465   0.000001465  -0.0000000   0.0        43
44      0.965665556  10.990000000   0.000001515   0.0000002  -0.0        44
45      0.930323350   0.000001547   0.000001547   0.0000000   0.0        45
46      0.816992827   0.000001920   0.000001215  -0.0000000   0.0        46
47      0.752734110   0.000001980   0.000000334   0.0000000   0.0        47
48      0.748833237  10.990000000   0.0          -0.0000000   0.0        48
49      1.050140264  10.990000000   0.000000000  -0.0000007  -0.0        49
50      1.047506295   0.000000979   0.000000228  -0.0000007   0.0        50
51      1.000010081   0.000000960   0.000000960  -0.0000000   0.0        51
52      0.898103994   0.000001052   0.000001602   0.0000000   0.0        52
53      0.859289834  10.990000000   0.000001771  -0.0000000  -0.0        53
54      0.616993942   0.000001215   0.000001920  -0.0000000   0.0        54
55      0.640335622   0.000002443   0.000002443   0.0000000   0.0        55
56      0.479941778   0.000005256   0.000001097  -0.0000000   0.0        56
57      0.466560287  10.990000000   0.0           0.0000000   0.0        57
58      1.002609424  10.990000000   0.000000000   0.0000003  -0.0        58
59      1.000010338   0.000000225   0.000000225   0.0000003   0.0        59
60      0.952514151   0.000000228   0.000000979   0.0000000   0.0        60
61      0.842965384   0.000000286   0.000001820  -0.0000000   0.0        61
62      0.797891619  10.990000000   0.000002102   0.0000000  -0.0        62
63      0.752735827   0.000000334   0.000001980   0.0000000   0.0        63
64      0.479942297   0.000001097   0.000005256  -0.0000000   0.0        64
65      0.054689489   0.000003486   0.000003486   0.0000000   0.0        65
66      0.011148970  10.990000000  -0.000000000  -0.0000000   0.0        66
67      0.997411265  -0.000000000  10.990000000  -0.0000144  -0.0000146  67
68      0.949880198   0.000000000  10.990000000  -0.0000001  -0.0000001  68
69      0.839623961  -0.000000000  10.990000000   0.0000000   0.0000001  69
70      0.748834991  -0.000000000  10.990000000  -0.0000000   0.0        70
71      0.466560835   0.000000000  10.990000000   0.0000000   0.0        71
72      0.011148999  -0.000000000  10.990000000  -0.0000000   0.0        72
```

TABLE F.7

CONFIGURATION TWO: CONCENTRATION RESULTS FOR DISPERSION COEFFICIENT=.0001075 SQ.CM/S AND N=5

INITIAL VISCOSITY OF THE OIL= 100.000 CP
KD= 0.0001075SQ.CM/S
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT= 25
TIME=50,000 S
DT=50,000 S

COLLOCATION POINT	PRESSURE (PSI)	VELOCITY IN X DIRECTION CM/S	VELOCITY IN Y DIRECTION CM/S	CONCENTRATION		COLLOCATION POINT
				IMPLICIT METHOD	R-K METHOD	
1	2.395794653	0.000000000	10.990000000	0.0461469	0.0458263	1
2	1.893573156	-0.000000000	10.990000000	0.0003249	0.0003564	2
3	1.514036351	0.000000000	10.990000000	-0.0000342	-0.0000454	3
4	1.325556066	-0.000000000	10.990000000	0.0000205	0.0000282	4
5	1.223481067	0.000000000	10.990000000	-0.0000085	-0.0000112	5
6	1.185775722	-0.000000000	10.990000000	0.0000026	-0.0	6
7	1.119491111	0.000000000	10.990000000	-0.0000005	-0.0	7
8	1.050122807	0.000000000	10.990000000	0.0000002	-0.0	8
9	1.010663730	-0.000000000	10.990000000	-0.0000001	-0.0	9
10	1.000440693	-0.000000000	10.990000000	0.0000000	-0.0	10
11	2.395794659	10.990000000	0.000000000	0.0461469	0.0458263	11
12	2.352402911	0.000008264	0.000008264	0.0412828	0.0410013	12
13	1.882020956	0.000002263	0.000011834	0.0002907	0.0003187	13
14	1.512347532	0.000000356	0.000003351	-0.0000306	-0.0000406	14
15	1.324348961	0.000000246	0.000003566	0.0000184	0.0000252	15
16	1.223057498	0.000000094	0.000001829	-0.0000076	-0.0000100	16
17	1.204854747	10.990000000	0.000002122	0.0003805	0.0003737	17
18	1.185160689	0.000000128	0.000001993	0.0000023	0.0	18
19	1.118988901	0.000000105	0.000001547	-0.0000004	0.0	19
20	1.049672205	0.000000094	0.000000985	0.0000001	0.0	20
21	1.010223472	0.000000092	0.000000453	-0.0000000	0.0	21
22	1.000000000	0.000000092	0.000000092	0.0000000	0.0	22
23	0.999558146	10.990000000	0.000000000	-0.0000121	-0.0000121	23
24	1.893573310	10.990000000	-0.000000000	0.0003249	0.0003564	24
25	1.882021108	0.000011834	0.000002263	0.0002907	0.0003187	25
26	1.721273002	0.000005123	0.000005123	0.0000018	0.0000015	26
27	1.473274547	0.000001655	0.000003699	-0.0000003	-0.0000003	27
28	1.300976262	0.000000938	0.000002699	0.0000001	0.0000001	28
29	1.210206070	0.000000608	0.000002132	-0.0000001	-0.0000001	29
30	1.190378752	10.990000000	0.000002017	0.0000027	0.0000029	30
31	1.171585111	0.000000583	0.000001909	0.0000000	0.0	31
32	1.107509133	0.000000502	0.000001512	-0.0000000	0.0	32
33	1.039267538	0.000000459	0.000000976	0.0000000	0.0	33

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34     1.000008814   0.000000452   0.000000452  -0.0000000   0.0          34
35     0.989789833   0.000000452   0.000000092   0.0000000   0.0          35
36     0.989349338  10.990000000   0.0          -0.0000001  -0.0000001    36
37     1.514037077  10.990000000  -0.000000000  -0.0000342  -0.0000454    37
38     1.512348252   0.000003351   0.000000356  -0.0000306  -0.0000406    38
39     1.473275116   0.000003699   0.000001655  -0.0000003  -0.0000003    39
40     1.356043675   0.000002354   0.000002354   0.0000000   0.0000000    40
41     1.232651510   0.000001477   0.000002083  -0.0000000  -0.0000000    41
42     1.158569700   0.000001205   0.000001861   0.0000000   0.0000000    42
43     1.141111357  10.990000000   0.000001781  -0.0000003  -0.0000004    43
44     1.124405368   0.000001101   0.000001709  -0.0000000   0.0          44
45     1.065669113   0.000001001   0.000001419   0.0000000   0.0          45
46     1.000013753   0.000000962   0.000000962  -0.0000000   0.0          46
47     0.960756949   0.000000976   0.000000459   0.0000000   0.0          47
48     0.950351500   0.000000985   0.000000094  -0.0000000   0.0          48
49     0.949900948  10.990000000   0.000000000   0.0000000   0.0000000    49
50     1.325557798  10.990000000   0.000000000   0.0000205   0.0000282    50
51     1.324350686   0.000003566   0.000000246   0.0000184   0.0000252    51
52     1.300977821   0.000002699   0.000000938   0.0000001   0.0000001    52
53     1.232652462   0.000002083   0.000001477  -0.0000000  -0.0000000    53
54     1.145348172   0.000001643   0.000001643   0.0000000   0.0000000    54
55     1.084449507   0.000001450   0.000001586  -0.0000000  -0.0000000    55
56     1.069376634  10.990000000   0.000001562   0.0000002   0.0000002    56
57     1.054567129   0.000001404   0.000001531   0.0000000   0.0          57
58     1.000016304   0.000001365   0.000001365  -0.0000000   0.0          58
59     0.934361541   0.000001419   0.000001001   0.0000000   0.0          59
60     0.892520005   0.000001512   0.000000502  -0.0000000   0.0          60
61     0.881039932   0.000001547   0.000000105   0.0000000   0.0          61
62     0.880537787  10.990000000  -0.000000000  -0.0000000  -0.0000000    62
63     1.223483761  10.990000000  -0.000000000  -0.0000085  -0.0000112    63
64     1.223060182   0.000001829   0.000000094  -0.0000076  -0.0000100    64
65     1.210208558   0.000002132   0.000000608  -0.0000001  -0.0000001    65
66     1.158571503   0.000001861   0.000001205   0.0000000   0.0000000    66
67     1.084450298   0.000001586   0.000001450  -0.0000000  -0.0000000    67
68     1.028719557   0.000001505   0.000001505   0.0000000   0.0000000    68
69     1.014327392  10.990000000   0.000001499  -0.0000001  -0.0000001    69
70     1.000017639   0.000001489   0.000001489  -0.0000000   0.0          70
71     0.945466957   0.000001531   0.000001404   0.0000000   0.0          71
72     0.875627286   0.000001709   0.000001101  -0.0000000   0.0          72
73     0.828446585   0.000001909   0.000000583   0.0000000   0.0          73
74     0.814871028   0.000001993   0.000000128  -0.0000000   0.0          74
75     0.814256082  10.990000000   0.000000000   0.0000000   0.0000000    75
76     1.204857717   0.000002122  10.990000000   0.0003205   0.0003737    76
77     1.190381512   0.000002017  10.990000000   0.0000027   0.0000029    77
78     1.141113403   0.000001781  10.990000000  -0.0000003  -0.0000004    78
79     1.069377639   0.000001562  10.990000000   0.0000002   0.0000002    79
80     1.014327600   0.000001499  10.990000000  -0.0000001  -0.0000001    80
81     0.985708205   0.000001499  10.990000000   0.0000000  -0.0          81
82     0.930657831   0.000001562  10.990000000  -0.0000000  -0.0          82
83     0.858921785   0.000001781  10.990000000   0.0000000  -0.0          83
84     0.809653523   0.000002017  10.990000000  -0.0000000  -0.0          84
85     0.795177506   0.000002122  10.990000000   0.0000000  -0.0          85
86     1.185778962  10.990000000   0.000000000   0.0000026  -0.0          86
87     1.185163923   0.000001993   0.000000128   0.0000023   0.0          87
88     1.171588159   0.000001909   0.000000583   0.0000000   0.0          88
89     1.124407667   0.000001709   0.000001101  -0.0000000   0.0          89
90     1.054568353   0.000001531   0.000001404   0.0000000   0.0          90
91     1.000018055   0.000001489   0.000001489  -0.0000000   0.0          91
92     0.985708412  10.990000000   0.000001499   0.0000000  -0.0          92
93     0.971316162   0.000001505   0.000001505   0.0000000   0.0          93
```

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```
 94          0.915585137     0.000001586     0.000001450    -0.0000000     0.0           94
 95          0.841463719     0.000001861     0.000001205     0.0000000     0.0           95
 96          0.789826564     0.000002132     0.000000608    -0.0000000     0.0           96
 97          0.776975145     0.000001829     0.000000094     0.0000000     0.0           97
 98          0.776551658    10.990000000     0.0            -0.0000000     0.0           98
 99          1.119495831    10.990000000    -0.000000000    -0.0000005    -0.0           99
100          1.118993604     0.000001547     0.000000105    -0.0000004     0.0          100
101          1.107513497     0.000001512     0.000000502    -0.0000000     0.0          101
102          1.065672487     0.000001419     0.000001001     0.0000000     0.0          102
103          1.000018411     0.000001365     0.000001365    -0.0000000     0.0          103
104          0.945468181     0.000001404     0.000001531     0.0000000     0.0          104
105          0.930658837    10.990000000     0.000001562    -0.0000000    -0.0          105
106          0.915585928     0.000001450     0.000001586    -0.0000000     0.0          106
107          0.854687162     0.000001643     0.000001643     0.0000000     0.0          107
108          0.767382895     0.000002083     0.000001477    -0.0000000     0.0          108
109          0.699057603     0.000002699     0.000000938     0.0000000     0.0          109
110          0.675684997     0.000003566     0.000000246    -0.0000000     0.0          110
111          0.674477979    10.990000000    -0.000000000     0.0000000     0.0          111
112          1.050130260    10.990000000     0.000000000     0.0000002    -0.0          112
113          1.049679634     0.000000985     0.000000094     0.0000001     0.0          113
114          1.039274400     0.000000976     0.000000459     0.0000000     0.0          114
115          1.000018915     0.000000962     0.000000962    -0.0000000     0.0          115
116          0.934364915     0.000001001     0.000001419     0.0000000     0.0          116
117          0.875629585     0.000001101     0.000001709    -0.0000000     0.0          117
118          0.858923831    10.990000000     0.000001781     0.0000000    -0.0          118
119          0.841465522     0.000001205     0.000001861     0.0000000     0.0          119
120          0.767383847     0.000001477     0.000002083    -0.0000000     0.0          120
121          0.643991971     0.000002354     0.000002354     0.0000000     0.0          121
122          0.526760763     0.000003699     0.000001655    -0.0000000     0.0          122
123          0.487687932     0.000003351     0.000000356     0.0000000     0.0          123
124          0.485999204    10.990000000     0.000000000    -0.0000000     0.0          124
125          1.010676666    10.990000000     0.000000000    -0.0000001    -0.0          125
126          1.010236261     0.000000452     0.000000092    -0.0000000     0.0          126
127          1.000019272     0.000000452     0.000000452    -0.0000000     0.0          127
128          0.960763811     0.000000459     0.000000976     0.0000000     0.0          128
129          0.892524369     0.000000502     0.000001512    -0.0000000     0.0          129
130          0.828449633     0.000000583     0.000001909     0.0000000     0.0          130
131          0.809656284    10.990000000     0.000002017    -0.0000000    -0.0          131
132          0.789829051     0.000000608     0.000002132    -0.0000000     0.0          132
133          0.699059162     0.000000938     0.000002699     0.0000000     0.0          133
134          0.526761332     0.000001655     0.000003699    -0.0000000     0.0          134
135          0.278763207     0.000005123     0.000005123     0.0000000     0.0          135
136          0.118015436     0.000011834     0.000002263    -0.0000000     0.0          136
137          0.106463326    10.990000000     0.0             0.0000000     0.0          137
138          1.000460932    10.990000000     0.000000000     0.0000000    -0.0          138
139          1.000019610     0.000000092     0.000000092     0.0000000     0.0          139
140          0.989802622     0.000000092     0.000000452     0.0000000     0.0          140
141          0.950358929     0.000000094     0.000000985    -0.0000000     0.0          141
142          0.881044635     0.000000105     0.000001547     0.0000000     0.0          142
143          0.814874262     0.000000128     0.000001993    -0.0000000     0.0          143
144          0.795180476    10.990000000     0.000002122     0.0000000    -0.0          144
145          0.776977830     0.000000094     0.000001829     0.0000000     0.0          145
146          0.675686722     0.000000246     0.000003566    -0.0000000     0.0          146
147          0.487688653     0.000000356     0.000003351     0.0000000     0.0          147
148          0.118015584     0.000002263     0.000011834    -0.0000000     0.0          148
149         -0.352366035     0.000008264     0.000008264     0.0000000     0.0          149
150         -0.395757686    10.990000000     0.0            -0.0000000     0.0          150
151          0.999578385    -0.000000000    10.990000000    -0.0000121    -0.0000121   151
152          0.989362294     0.0            10.990000000    -0.0000001    -0.0000001   152
153          0.949908402     0.0            10.990000000     0.0000000     0.0000000   153
154          0.880542507     0.0            10.990000000    -0.0000000    -0.0000000   154
155          0.814259322     0.000000000    10.990000000     0.0000000     0.0000000   155
156          0.776554352     0.0            10.990000000    -0.0000000     0.0          156
157          0.674479711    -0.000000000    10.990000000     0.0000000     0.0          157
158          0.485999931     0.000000000    10.990000000    -0.0000000     0.0          158
159          0.106463481     0.0            10.990000000     0.0000000     0.0          159
160         -0.395757679     0.000000000    10.990000000    -0.0000000     0.0          160
```

TABLE F.8

CONFIGURATION TWO: CONCENTRATION RESULTS FOR DISPERSION COEFFICIENT=.01075 SQ.CM/S AND N=5

```
INITIAL VISCOSITY OF THE OIL=   100.000 CP
KD=     0.0107500SQ.CM/S
NUMBER OF INTERIOR COLLOCATION POINTS PER ELEMENT= 25
TIME=50,000 S
DT=50,000 S


COLLOCATION     PRESSURE        VELOCITY IN      VELOCITY IN           CONCENTRATION            COLLOCATION
  POINT          (PSI)          X DIRECTION      Y DIRECTION                                       POINT
                                  CM/S           CM/S              IMPLICIT       R-K
                                                                    METHOD       METHOD


    1          2.395794653      0.000000000     10.990000000      0.0430041    0.0367055     1
    2          1.893573156     -0.000000000     10.990000000      0.0011344    0.0009275     2
    3          1.514036351      0.000000000     10.990000000     -0.0001609   -0.0001909     3
    4          1.325556066     -0.000000000     10.990000000      0.0000983    0.0001381     4
    5          1.223481067      0.000000000     10.990000000     -0.0001152   -0.0002543     5
    6          1.185775722     -0.000000000     10.990000000      0.0000465   -0.0           6
    7          1.119491111      0.000000000     10.990000000     -0.0000025   -0.0           7
    8          1.050122807      0.000000000     10.990000000      0.0000010   -0.0           8
    9          1.010663730     -0.000000000     10.990000000     -0.0000008   -0.0           9
   10          1.000440693     -0.000000000     10.990000000      0.0000006   -0.0          10
   11          2.395794659     10.990000000      0.000000000      0.0430041    0.0367055    11
   12          2.352402911      0.000008264      0.000008264      0.0385935    0.0329442    12
   13          1.882020956      0.000002263      0.000011834      0.0010212    0.0008329    13
   14          1.512347532      0.000000356      0.000003351     -0.0001449   -0.0001715    14
   15          1.324348961      0.000000246      0.000003566      0.0000884    0.0001240    15
   16          1.223057498      0.000000094      0.000001829     -0.0001036   -0.0002284    16
   17          1.204854747     10.990000000      0.000002122      0.0002983    0.0001515    17
   18          1.185160689      0.000000128      0.000001993      0.0000418    0.0          18
   19          1.118988901      0.000000105      0.000001547     -0.0000022    0.0          19
   20          1.049672205      0.000000094      0.000000985      0.0000009    0.0          20
   21          1.010223472      0.000000092      0.000000453     -0.0000007     .0          21
   22          1.000000000      0.000000092      0.000000092      0.0000006    0.0          22
   23          0.999558146     10.990000000      0.000000000     -0.0000064   -0.0000049    23
   24          1.893573310     10.990000000     -0.000000000      0.0011344    0.0009275    24
   25          1.882021104      0.000011834      0.000002263      0.0010212    0.0008329    25
   26          1.721273002      0.000005123      0.000005123      0.0000463    0.0000234    26
   27          1.473274547      0.000001653      0.000003699     -0.0000066   -0.0000050    27
   28          1.300976262      0.000000938      0.000002699      0.0000038    0.0000035    28
   29          1.210206070      0.000000608      0.000002132     -0.0000044   -0.0000063    29
   30          1.190378752     10.990000000      0.000002017      0.0000066    0.0000034    30
   31          1.171565111      0.000000583      0.000001909      0.0000016    0.0          31
   32          1.107509133      0.000000502      0.000001512     -0.0000001    0.0          32
   33          1.039267538      0.000000459      0.000000976      0.0000000    0.0          33
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34      1.000008814    0.000000452    0.000000452   -0.0000000    0.0          34
35      0.989789833    0.000000452    0.000000092    0.0000000    0.0          35
36      0.989349338   10.990000000    0.0           -0.0000001   -0.0000001    36
37      1.514037677   10.990000000   -0.000000000   -0.0001609   -0.0001909    37
38      1.512348252    0.000003351    0.000000356   -0.0001449   -0.0001715    38
39      1.473275116    0.000003699    0.000001655   -0.0000066   -0.0000050    39
40      1.356043675    0.000002354    0.000002354    0.0000009    0.0000011    40
41      1.232651510    0.000001477    0.000002083   -0.0000006   -0.0000008    41
42      1.158569700    0.000001205    0.000001861    0.0000006    0.0000014    42
43      1.141111357   10.990000000    0.000001781   -0.0000009   -0.0000006    43
44      1.124405368    0.000001101    0.000001709   -0.0000002    0.0          44
45      1.065665113    0.000001001    0.000001419    0.0000000    0.0          45
46      1.000013753    0.000000962    0.000000962   -0.0000000    0.0          46
47      0.960756949    0.000000976    0.000000459    0.0000000    0.0          47
48      0.950351500    0.000000985    0.000000094   -0.0000000    0.0          48
49      0.949900948   10.990000000    0.000000000    0.0000000    0.0000000    49
50      1.325557798   10.990000000    0.000000000    0.0000983    0.0001381    50
51      1.324350686    0.000003566    0.000000246    0.0000884    0.0001240    51
52      1.300977821    0.000002699    0.000000938    0.0000038    0.0000035    52
53      1.232652462    0.000002083    0.000001477   -0.0000006   -0.0000008    53
54      1.145348172    0.000001643    0.000001643    0.0000003    0.0000005    54
55      1.084449507    0.000001450    0.000001586   -0.0000004   -0.0000010    55
56      1.069376634   10.990000000    0.000001562    0.0000006    0.0000005    56
57      1.054567129    0.000001404    0.000001531    0.0000001    0.0          57
58      1.000016304    0.000001365    0.000001365   -0.0000000    0.0          58
59      0.934361541    0.000001419    0.000001001    0.0000000    0.0          59
60      0.892520005    0.000001512    0.000000502   -0.0000000    0.0          60
61      0.881039932    0.000001547    0.000000105    0.0000000    0.0          61
62      0.880537787   10.990000000   -0.000000000   -0.0000000   -0.0000000    62
63      1.223483761   10.990000000   -0.000000000   -0.0001152   -0.0002543    63
64      1.223060182    0.000001829    0.000000094   -0.0001036   -0.0002284    64
65      1.210208558    0.000002132    0.000000608   -0.0000044   -0.0000063    65
66      1.158571503    0.000001861    0.000001205    0.0000006    0.0000014    66
67      1.084450298    0.000001586    0.000001450   -0.0000004   -0.0000010    67
68      1.028719557    0.000001505    0.000001505    0.0000005    0.0000018    68
69      1.014327392   10.990000000    0.000001499   -0.0000007   -0.0000009    69
70      1.000017639    0.000001489    0.000001489   -0.0000002    0.0          70
71      0.945466957    0.000001531    0.000001404    0.0000000    0.0          71
72      0.875627286    0.000001709    0.000001101   -0.0000000    0.0          72
73      0.828446585    0.000001909    0.000000583    0.0000000    0.0          73
74      0.814871028    0.000001993    0.000000128   -0.0000000    0.0          74
75      0.814256082   10.950000000    0.000000000    0.0000000    0.0000000    75
76      1.204857717    0.000002122   10.990000000    0.0002983    0.0001515    76
77      1.190381512    0.000002017   10.990000000    0.000006[illegible]    0.0000034    77
78      1.141113403    0.000001781   10.990000000   -0.0000009   -0.0000006    78
79      1.069377639    0.000001562   10.990000000    0.0000006    0.0000005    79
80      1.014327600    0.000001499   10.990000000   -0.0000007   -0.0000009    80
81      0.985708205    0.000001499   10.990000000    0.0000003   -0.0          81
82      0.930657831    0.000001562   10.990000000   -0.0000000   -0.0          82
83      0.858921785    0.000001781   10.990000000    0.0000000   -0.0          83
84      0.809653523    0.000002017   10.990000000   -0.0000000   -0.0          84
85      0.795177506    0.000002122   10.990000000    0.0000000   -0.0          85
86      1.185778962   10.990000000    0.000000000    0.0000465   -0.0          86
87      1.185163923    0.000001993    0.000000128    0.0000418    0.0          87
88      1.171568159    0.000001909    0.000000583    0.0000016    0.0          88
89      1.124407667    0.000001709    0.000001101   -0.0000002    0.0          89
90      1.054568353    0.000001531    0.000001404    0.0000001    0.0          90
91      1.000018055    0.000001489    0.000001489   -0.0000002    0.0          91
92      0.985703412   10.990000000    0.000001499    0.0000003   -0.0          92
93      0.971316162    0.000001505    0.000001505    0.0000001    0.0          93
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CONTINUED

 94     0.915585137    0.000001586    0.000001450   -0.0000000    0.0           94
 95     0.841463719    0.000001861    0.000001205    0.0000000    0.0           95
 96     0.789826564    0.000002132    0.000000608   -0.0000000    0.0           96
 97     0.776975145    0.000001829    0.000000094    0.0000000    0.0           97
 98     0.776551658   10.990000000    0.0           -0.0000000    0.0           98
 99     1.119495831   10.990000000   -0.000000000   -0.0000025   -0.0           99
100     1.118993604    0.000001547    0.000000105   -0.0000022    0.0          100
101     1.107513497    0.000001512    0.000000502   -0.0000001    0.0          101
102     1.065672487    0.000001419    0.000001001    0.0000000    0.0          102
103     1.000018411    0.000001365    0.000001365   -0.0000000    0.0          103
104     0.945468181    0.000001404    0.000001531    0.0000000    0.0          104
105     0.930658837   10.990000000    0.000001562   -0.0000000   -0.0          105
106     0.915585928    0.000001450    0.000001586   -0.0000000    0.0          106
107     0.854687162    0.000001643    0.000001643    0.0000000    0.0          107
108     0.767382895    0.000002083    0.000001477   -0.0000000    0.0          108
109     0.699057603    0.000002699    0.000000938    0.0000000    0.0          109
110     0.675684997    0.000003566    0.000000246   -0.0000000    0.0          110
111     0.674477979   10.990000000   -0.000000000    0.0000000    0.0          111
112     1.050130260   10.990000000    0.000000000    0.0000010   -0.0          112
113     1.049679634    0.000000985    0.000000094    0.0000009    0.0          113
114     1.039274400    0.000000976    0.000000459    0.0000000    0.0          114
115     1.000018915    0.000000962    0.000000962   -0.0000000    0.0          115
116     0.934364915    0.000001001    0.000001419    0.0000000    0.0          116
117     0.875629585    0.000001101    0.000001709   -0.0000000    0.0          117
118     0.858923831   10.990000000    0.000001781    0.0000000   -0.0          118
119     0.841465522    0.000001205    0.000001861    0.0000000    0.0          119
120     0.767383847    0.000001477    0.000002083   -0.0000000    0.0          120
121     0.643991971    0.000002354    0.000002354    0.0000000    0.0          121
122     0.526760763    0.000003699    0.000001655   -0.0000000    0.0          122
123     0.467687932    0.000003351    0.000000356    0.0000000    0.0          123
124     0.485999204   10.990000000    0.000000000   -0.0000000    0.0          124
125     1.010676686   10.990000000    0.000000000   -0.0000008   -0.0          125
126     1.010236261    0.000000452    0.000000092   -0.0000007    0.0          126
127     1.000019272    0.000000452    0.000000452   -0.0000000    0.0          127
128     0.950763811    0.000000459    0.000000976    0.0000000    0.0          128
129     0.892524369    0.000000502    0.000001512   -0.0000000    0.0          129
130     0.828449633    0.000000583    0.000001909    0.0000000    0.0          130
131     0.809656284   10.990000000    0.000002017   -0.0000000   -0.0          131
132     0.789829051    0.000000608    0.000002132   -0.0000000    0.0          132
133     0.699059162    0.000000938    0.000002699    0.0000000    0.0          133
134     0.526761332    0.000001655    0.000003699   -0.0000000    0.0          134
135     0.278763207    0.000005123    0.000005123    0.0000000    0.0          135
136     0.118015436    0.000011834    0.000002263   -0.0000000    0.0          136
137     0.106463326   10.990000000    0.0            0.0000000    0.0          137
138     1.000460932   10.990000000    0.000000000    0.0000006   -0.0          138
139     1.000019610    0.000000092    0.000000092    0.0000006    0.0          139
140     0.989802622    0.000000092    0.000000452    0.0000000    0.0          140
141     0.950358929    0.000000094    0.000000985   -0.0000000    0.0          141
142     0.881044635    0.000000105    0.000001547    0.0000000    0.0          142
143     0.814874262    0.000000128    0.000001993   -0.0000000    0.0          143
144     0.795180476   10.990000000    0.000002122    0.0000000   -0.0          144
145     0.776977830    0.000000094    0.000001929    0.0000000    0.0          145
146     0.675686722    0.000000246    0.000003566   -0.0000000    0.0          146
147     0.487688653    0.000000356    0.000003351    0.0000000    0.0          147
148     0.118015584    0.000002263    0.000011838   -0.0000000    0.0          148
149    -0.352366035    0.000008264    0.000008264    0.0000000    0.0          149
150    -0.395757686   10.990000000    0.0           -0.0000000    0.0          150
151     0.999578385   -0.000000000   10.990000000   -0.0000064   -0.0000049   151
152     0.989362294    0.0           10.990000000   -0.0000001   -0.0000001   152
153     0.949908402    0.0           10.990000000    0.0000000    0.0000000   153
154     0.880542507    0.0           10.990000000   -0.0000000   -0.0000000   154
155     0.814259322    0.000000000   10.990000000    0.0000000    0.0000000   155
156     0.776554352    0.0           10.990000000   -0.0000000    0.0          156
157     0.674479711   -0.000000000   10.990000000    0.0000000    0.0          157
158     0.485999931    0.000000000   10.990000000   -0.0000000    0.0          158
159     0.106463481    0.0           10.990000000    0.0000000    0.0          159
160    -0.395757679    0.000000000   10.990000000   -0.0000000    0.0          160
```

APPENDIX G

Hand Calculations for Concentration Profiles

This appendix contains hand calculations for two cases for the convection-diffusion equation (for details see Section 5.7.2).

Two extreme cases are considered. Values of concentration are checked only for the Total Implicit method. For N=3 the concentration values are taken from Table F.2, and for N=5, from Table F.4.

Case 1 (For N=3)

The collocation point Number 10 is considered (see Figure 11 for numbering of unknowns). Equation (5.30) was used for the Total Implicit formulation. Concentration of the solvent at the t time level was assumed zero for the entire formation. Also for the collocation point, Number 10, q is zero. Therefore Equation (5.30) reduces to

$$\frac{C_{i,j}^{t+\Delta t,k,\ell}}{\Delta t} = P_1 \sum_{n=1}^{NPX} B_{i,n} C_{n,j}^{t+\Delta t,k,\ell} + P_2 \sum_{n=1}^{NPY} B_{j,n} C_{i,n}^{t+\Delta t,k,\ell} - P_3 \sum_{n=1}^{NPX} A_{i,n} C_{n,j}^{t+\Delta t,k,\ell} - P_4 \sum_{n=1}^{NPY} A_{j,n} C_{i,n}^{t+\Delta t,k,\ell} \tag{G.1}$$

The following collocation points are needed for the first and the second derivatives (Figure G.1, not to the scale).

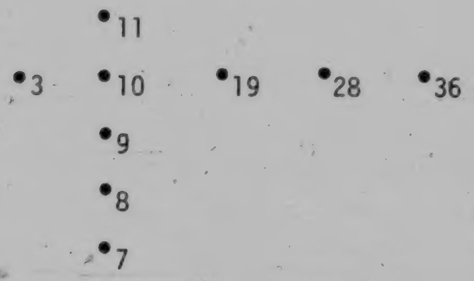

Figure G.1. Collocation points for first and second Derivatives for N=3.

P_1, P_2, P_3 and P_4 are given by Equation 5.26

$$P_1 = \frac{K_D}{L^2 \phi \Delta x_k{}^2} = \frac{.01075}{(6000)^2 x\ .1x(.5)} = 11.944 \times 10^{-9}$$

$$P_2 = \frac{K_D}{W^2 \phi \Delta y_1{}^2} = \frac{.01075}{(6000)^2 x\ .1x(.5)^2} = 11.944 \times 10^{-9}$$

$$P_3 = \frac{U_x^{k,1}\ (x_i, y_j)}{L \phi \Delta x_k} = \frac{3.53 \times 10^{-7}}{6000 \times .1x.5} = 11.766 \times 10^{-10}$$

$$P_4 = \frac{U_y^{k,1}\ (x_i, y_j)}{W \phi \Delta y_1} = \frac{1.36 \times 10^{-6}}{6000 \times .1x.5} = 4.533 \times 10^{-9}$$

Summations in Equation (G.1) are expanded below

$$\sum_{n=1}^{NPX} B_{i,n}\ C_{n,j}^{k,\ell} = B_{2,1}C_3 + B_{2,2}\ C_{10} + B_{2,3}\ C_{19} + B_{2,4}C_{28} + B_{2,5}C_{36}$$

$$\sum_{n=1}^{NPX} A_{i,n}\ C_{n,j}^{k,\ell} = A_{2,1}C_3 + A_{2,2}\ C_{10} + A_{2,3}\ C_{19} + A_{2,4}C_{28}\ \ A_{2,5}C_{36}$$

$$\sum_{n=1}^{NPY} B_{j,n}\ C_{i.n}^{k,\ell} = B_{4,1}\ C_7 + B_{4,2}\ C_8 + B_{4,3}\ C_9 + B_{4,4}\ C_{10} + B_{4,5}\ C_{11}$$

$$\sum_{n=1}^{NPY} A_{j,n}\ C_{i,n}^{k,\ell} = A_{4,1}\ C_7 + A_{4,2}\ C_8 + A_{4,3}\ C_9 + A_{4,4}\ C_{10} + A_{4,5}\ C_{11}$$

where C_i is the concentration at the i^{th} collocation point.

The numerical values of the summations can be evaluated as follows:

$$\begin{aligned}\sum_{n=1}^{NPX} B_{i,n}\ C_{n,j}^{k,\ell} &= 53.2379 \times -.0000209 - 73.333333 \times -.0000185 \\ &\quad + 26.666667 \times -.0000002 - 13.333333 \times .0000001 \\ &\quad + 6.7621 \times -.0000005 \\ &= .0002339468\end{aligned}$$

$$\sum_{n=1}^{NPX} A_{i,n}\, C^{k,\ell}_{n,j} = -5.32379 \times -.0000209 + 3.872983 \times -.0000185$$
$$+ 2.065591 \times -.0000002 - 1.290994 \times .0000001$$
$$+ .67621 \times -.0000005$$
$$= .0000387367$$

$$\sum_{n=1}^{NPY} B_{j,n}\, C^{k,\ell}_{i,n} = 6.7621 \times .0086014 - 13.333333 \times .007587$$
$$+ 26.666667 \times .0000523 - 73.333333 \times -.0000185$$
$$+ 53.23790 \times .0002057$$
$$= -.0292941012$$

$$\sum_{n=1}^{NPY} A_{j,n}\, C^{k,\ell}_{i,n} = -.67621 \times .0086014 + 1.290994 \times .007587$$
$$-2.065591 \times .0000523 - 3.872983 \times -.0000185$$
$$+ 5.32379 \times .0002057$$
$$= .0050371422$$

Substituting the values of the summations in Equation G.1 yields

$$\frac{C^{t+\Delta t,k,\ell}_{i,j}}{\Delta t} = 11.944 \times 10^{-9} \times .0002339468 + 11.944 \times 10^{-9} \times -.0292941012$$
$$- 11.766 \times 10^{-10} \times .0000387367 - 4.533 \times 10^{-9} \times .0050371422$$
$$= -.3559433501 \times 10^{-9}$$

$$C_{10} = -.3554433501 \times 10^{-9} \times 50000$$
$$= -1.77 \times 10^{-5}$$

The corresponding value in Table F.2 for the Total Implicit method is -1.85×10^{-5}.

Case 2 (For N=5)

Collocation point Number 27 is considered (see Figure 13 for numbering of unknowns). Equation G.1 still holds with different values for NPX and NPY.

The following collocation points are needed for the first and the second derivatives (Figure G.2, not to the scale).

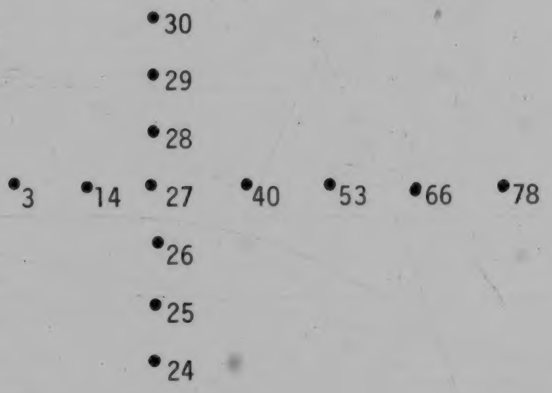

Figure G.2. Collocation points for first and second Derivatives for N=5.

The values of P_1, P_2, P_3 and P_4 are given by

$$P_1 = P_2 = 11.944 \times 10^{-9}$$

$$P_3 = \frac{U_x^{k,\ell}(x_i,y_j)}{L\phi\Delta x_k} = \frac{.000001817}{6000 \text{ x}.1\text{x}.5} = 6.0566 \times 10^{-9}$$

$$P_4 = \frac{U_y^{k,\ell}(x_i,y_j)}{W\phi\Delta y_1} = \frac{.000003230}{6000 \text{ x}.1\text{x}.5} = 10.7666 \times 10^{-9}$$

Summations in Equation G.1 are expanded below:

$$\sum_{n=1}^{NPX} B_{i,n} C^{k,\ell}_{n,j} = B_{3,1}C_3 + B_{3,2}C_{14} + B_{3,3}C_{27} + B_{3,4}C_{40} + B_{3,5}C_{53} + B_{3,6}C_{66} + B_{3,7}C_{78}$$

$$\sum_{n=1}^{NPX} A_{i,n} C^{k,\ell}_{n,j} = A_{3,1}C_3 + A_{3,2}C_{14} + A_{3,3}C_{27} + A_{3,4}C_{40} + A_{3,5}C_{53} + A_{3,6}C_{66} + A_{3,7}C_{78}$$

$$\sum_{n=1}^{NPY} B_{j,n} C^{k,\ell}_{i,n} = B_{4,1}C_{24} + B_{4,2}C_{25} + B_{4,3}C_{26} + B_{4,4}C_{27} + B_{4,5}C_{28} + B_{4,6}C_{29} + B_{4,7}C_{30}$$

$$\sum_{n=1}^{NPY} A_{j,n} C^{k,\ell}_{i,n} = A_{4,1}C_{24} + A_{4,2}C_{25} + A_{4,3}C_{26} + A_{4,4}C_{27} + A_{4,5}C_{28} + A_{4,6}C_{29} + A_{4,7}C_{30}$$

Therefore the numerical values of the summations can be evaluated as follows:

$$\begin{aligned}\sum_{n=1}^{NPX} B_{i,n} C^{k,\ell}_{n,j} &= -21.024726 \times -.0001575 + 59.816814 \times -.0001418 \\ &\quad -66.912395 \times -.0000064 + 35.697494 \times .0000009 \\ &\quad -12.531160 \times -.0000006 + 11.261251 \times .0000006 \\ &\quad -6.307280 \times -.0000009 \\ &= -.0046903108\end{aligned}$$

$$\begin{aligned}\sum_{n=1}^{NPX} A_{i,n} C^{k,\ell}_{n,j} &= 3.732156 \times -.0001575 - 7.625116 \times -.0001418 \\ &\quad +1.516706 \times -.0000064 + 3.412150 \times .0000009 \\ &\quad -1.857116 \times -.0000007 + 1.940842 \times .0000006 \\ &\quad -1.119622 \times -.0000009 \\ &= .000490073\end{aligned}$$

$$\sum_{n=1}^{NPY} B_{j,n}\, C_{i,n}^{k,\ell} = 7.5 \times .0011397 - 14.867044 \times .0010258$$

$$+ 30.033711 \times .0000463 - 45.333333 \times - .0000064$$

$$+ 30.033711 \times .0000037 - 14.867044 \times -.0000043$$

$$+ 7.5 \times .0000067$$

$$= - .0047968666$$

$$\sum_{n=1}^{NPY} A_{j,n}\, C_{i,n}^{k,\ell} = - 1.875 \times .0011397 + 3.368054 \times .0010258$$

$$-4.043058 \times .0000463 + 0 \times - .0000064$$

$$+4.043057 \times .0000037 - 3.368054 \times - .0000043$$

$$+1.875 \times .0000067$$

$$= 0.0011728232$$

Substituting the values of the summations in Equation G.1 yields

$$\frac{C_{i,j}^{t+\Delta t,k,\ell}}{\Delta t} = 11.944 \times 10^{-9} \, [- .0046903108 - .0047968666]$$

$$-6.0566 \times 10^{-9} \times .0004900773 - 10.766 \times 10^{-9} \times .0011728232$$

$$= - .12891036 \times 10^{-9}$$

$$\therefore C_{27} = - .1289103673 \times 10^{-9} \times 50000$$

$$C_{27} = - 6.44 \times 10^{-6}$$

The corresponding value in Table F.4 is -6.4×10^{-6}

APPENDIX H

Physical Data for Porous Media Problem

1. Injection rate of the solvent Q, 0.3277 cm^3/s
2. Viscosity of the injected solvent μ_s, 1 cp
3. Porosity of the formation , 0.1
4. Permeability of the formation K_p, 0.1 darcy
5. Length of the formation L, 6000 cm
6. Width of the formation W, 6000 cm
7. Thickness of the formation S, 30.48 cm
8. Initial viscosity of the oil in the formation μ, 100 and 10000 cp
9. Dispersion Coefficient K_D, 0.0001075 and 0.01075 cm^2/s
10. Δt, 50,000 s
11. Pressure at the reference collocation point, 1 psi.

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